BOUNDARY LAYER PROFILE
BACKGROUND AND MATHEMATICAL TOOLS

J. Andrew Breuer
Consultant

Joseph H. Rousseau, James N. Brekke
AMERICAN BUREAU OF SHIPPING

Abstract

The Offshore Committee of Society of Naval Architects and Marine Engineers is engaged in a number of initiatives associated with the technology of determining wind forces and moments. This includes a revision to the *Guidelines for Wind Tunnel Testing of Mobile Offshore Drilling Units,* the use of computational fluid dynamics (CFD), and the development of empirical tools. The purpose of this initiative is to vet these new technologies so they can be incorporated into the regulatory approval procedures by class societies and flag and coastal regulatory agencies.

The wind boundary layer profile has been expressed in a number of formulations. Of these, the power law profile formulation is the one that has been used most often in the design of jack-up rigs and other offshore units. This paper presents the history of the boundary layer profile, how it was incorporated into existing standards, and closed-form mathematical equations needed to calculate wind forces and moments with a method acceptable to regulatory agencies. These closed-form equations are intended to update the process of empirical calculations, commonly used in the design spiral, to a level consistent with modern technology.

Key Words: Wind, Environment, Loads

Introduction

While the drama at Macondo dominated the news and all the eyes of our Industry were fixated on it in 2010, a quiet discussion evolved in the world of offshore regulations. At the center of this was the US Coast Guard ("USCG") negative judgment on the suitability of wind tunnel testing results for the purpose of regulatory approval of offshore assets under Title 46 of the US Code of Federal Regulations.[1]

The main concern at the USCG is the lack of clear guidance of what constitutes a bonafide wind tunnel test. Informally, the USCG indicated that guidance developed under the auspices of the Society of Naval Architects and Marine Engineers (SNAME) could lead to an acceptable standard. The SNAME Offshore Committee re-activated their Stability and Motions Panel (OC-1) to lead such effort.

From the onset, the Committee recognized the need for an update to the SNAME Guidance on the subject[2] . It also raised the question on the outdated methods for wind force calculations developed in the mid '60s and the lack of an update for Computational Fluid Dynamics (CFD) within the regulatory guidance.

This paper serves to propose a better methodology than the fifty-year old empirical formula published in the ABS Rules for Building and Classing Mobile Offshore Drilling Units, 1968 (MODU Rules):[3]

$$F = 0.00338 \times V^2 \times Ch \times Cs$$

The new methodology is clear but its execution is complex and must be implemented with a computer. This paper presents a means for successfully resolving this challenge. Worth noting is that a few offshore regulations have stepped back to the fundamentals. However, relevant regulators such as the USCG, Canadian provincial regulations, and the IMO[4] have retained the old methodology. Additionally, this paper proposes an alternative approach that should prove to be equivalent or more conservative and that should allow these organizations to apply the methodology in lieu of the currently published.

Part of the development of the original methodology was the accounting for the boundary layer effect and the increase in wind velocity with the rise in elevation. The application of a Height Coefficient (*Ch)* to correct the wind pressures to account for the higher wind velocity with the increase of elevation resolved this matter. The resulting table of *Ch* is reflected in the illustration. The earliest this subject was addressed is found in a USCG published Marine Notice 6-66.[5] This document proposed a Power-Law Boundary Layer and an equivalent table. The Power-Law Boundary Layer Profile was considered during the original MODU Rule development; for practical purposes, the table was adopted although without a reference to its source. The method simplifies the continuous increase of velocity as a discontinuous function of elevation by adopting a regular step increase at every 50 foot increase in elevation. This distribution is frequently referred to as the "step function".

§ 174.065

TABLE 174.055(a)—CH VALUES

Feet		Meters		Ch.
Over	Not exceeding	Over	Not exceeding	
0	50	0.0	15.3	1.00
50	100	15.3	30.5	1.10
100	150	30.5	46.0	1.20
150	200	46.0	61.0	1.30
200	250	61.0	76.0	1.37
250	300	76.0	91.5	1.43
300	350	91.5	106.5	1.48
350	400	106.5	2.0	1.52
400	450	122.0	137.0	1.56
450	500	137.0	152.5	1.60
500	550	152.5	167.5	1.63
550	600	167.5	183.0	1.67
600	650	183.0	198.0	1.70
650	700	198.0	213.5	1.72
700	750	213.5	228.5	1.75
750	800	228.5	244.0	1.77
800	850	244.0	256.0	1.79
Above 850		Above 256		1.80

NOTE: The "Ch" value in this table, used in the equation described in section § 174.055(b), corresponds to the value of the vertical distance in feet (meters) from the water surface at the design draft of the unit to the center of area of the "A" value used in the equation.

Figure 1 - Table of Height Coefficients as reproduced in the US Code of Federal Regulations Title 46 - Subchapter S

The power-law $\frac{v}{V_r} = \left[\frac{z}{Z_r}\right]^{\propto}$ where

- v is the wind velocity at elevation z above the sea level;
- V_r is the nominal velocity established for the design (typically 100 knots);
- Zr is the nominal elevation at which the wind velocity Vr is established; and
- α is an exponent that usually ranges between $^1/_8$ and $^1/_{12}$ and changes depending on the local environmental conditions.

Thus, it may be deducted that

$$v(z) = V_r \left[\frac{z}{Z_r}\right]^{\alpha} \text{ and } v^2(z) = {V_r}^2 \left[\frac{z}{Z_r}\right]^{2\alpha}$$

Therefore, the relationship between the height coefficients – the ratio between v^2 and Vr^2 - and the profile is:

$$C_h = \left[\frac{z}{Z_r}\right]^{2\alpha}$$

Without a documented power-law profile, the value of the exponent and the reference elevation have been the subject of speculation; however, the conclusion of the authors is that the combination of $\alpha = 0.1$ a $Z_r = 50$ Ft, and selecting the C_h at each layer to correspond to the most conservative (and higher) value in the range are the most accurate reconstruction of the method.

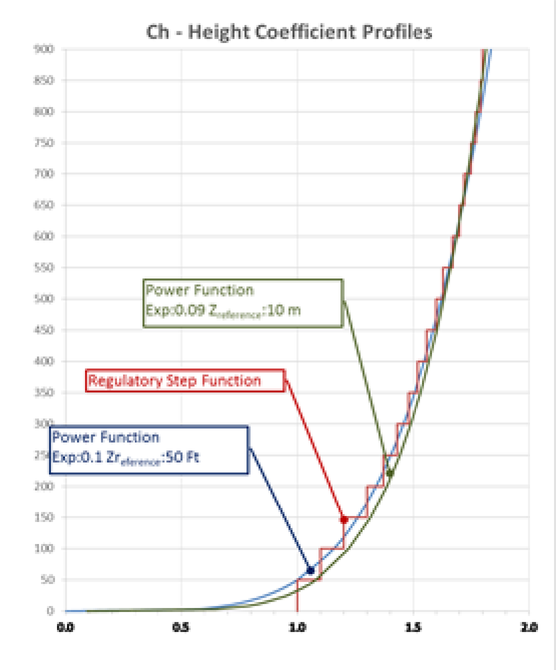


Figure 1 - comparison between Boundary Layer Profile Height Coefficients

The step function reflected in Table 1 is reproduced from the United States Code of Federal Regulations and any attempt to determine the value of the exponent α and elevation Zr will prove that there is no value that will result in a table of *Ch* that matches the published table exactly; round-off of the tabular values and possible numerical errors may be an explanation for the curve fitting problem. A least square curve determination of the parameters will prove that a value of

$\alpha = 0.105$ with a $Z_r = 50$ ft will provide the closest average wind force. These figures have been widely used at wind tunnel tests on offshore structures.

α = 0.09 with a Z_r = 50 ft will result in a very close match with each step in the *Ch* table.

α = 0.1 with a Z_r = 10m, a profile that matches published data by institutions like UK HSE, API, and the SNAME RP 5-5A[6], will result in mildly but consistently conservative results

Current Practice

The application of the current rules and regulations is very simple because when the original rules were written, the most frequent calculating tools available to designers were slide rules and the abacus. Hand held electronic calculators did not reach the market until the '70s.

Despite its simplicity, the ways these rules and regulations are applied vary from user to user. Some designers will determine the projected area of each exposed part and apply the C_h that corresponds to the centroid. Others will split the projected area into 50 foot (15.24m) strips and calculate each strip separately.

Some designers will use the half draft as the center of lateral resistance, while others will calculate the centroid of the effective area of the underwater body.

Such differences arise from the complexities, the tedious numerical exercise, and the absence of guidance, and lack of detail in the regulations.

Proposed methodology

This proposal is specifically devised for computer application; a major part of it is the incorporation of a power law boundary layer. This will lead to a more accurate, efficient process and should provide results acceptable to regulators that currently require the use of a step function as discussed above.

Fluid Drag Quadratic Equation.

It is easy to recognize the familiarity between wind force empirical equation in the current regulations and Fluid Drag Quadratic equation.

$$F = f v_k^2 C_h C_s A \approx 1/2 \rho v^2 C_d A$$

Where:

F: is the wind force or fluid drag force in N, Kg_f, or Lb_f;

f: is a constant, equivalent to the ½ *ρ* in Fluid Drag Quadratic equation;

A: is the area of the body projected on a surface perpendicular to the direction of wind;

Cs: is the shape coefficient, and accounts for the difference in geometrical form of the body expose. The coefficient is equivalent to the Drag coefficient Cd. However, Cd applies to simple geometrical elements, whereas Cs applies to composite forms and the interaction between the many components in an offshore facility.

P: is the density of air - approximately 1.225 kg/m^3, 0.0023769 $slug/ft^3$, or 0.0765 Lb_m/ft^3) according to ISA (International Standard Atmosphere);[7]

v: is the velocity of the fluid. While this velocity is assumed constant in the quadratic equation, the regulatory corrects for the increase in velocity with the rise in elevation with the use of the height coefficient Ch;

C_d: is the drag coefficient – a non-dimensional coefficient established on the basis of wind tunnel measurements that applies for simple forms usually of "infinite" length; and

C_h: is the height coefficient that accounts for the boundary layer effect.

The application of the Quadratic Equation, combined with a selected boundary layer, is complex enough that any reasonable "manual" methodology must sacrifice accuracy for the sake of expediency. The regulatory requirements are a clear example of how simplification ignored multiple parameters. Not only was the boundary layer effect s resolved by creating 50-foot layers; the simplifications are many to having them listed. Among these, we can mention the effect of shielding, the grouping of elements, the adoption of invariable shape coefficients, disregard to the effect of Reynolds Numbers, turbulence, lift, and the disturbed flow of wind around the installation.

While this complexity can only be resolved by high level technology, CFD is the most promising one. Though much work needs to be done before CFD is found to be a verified method that can replace the existing regulatory standards, until then the authors find that the methods may be upgraded to allow efficient use of computer programs while maintaining the goals of the current Rules. For this purpose, two things are needed:

1. An algorithm that scientifically combines the geometry of discrete geometrically defined components; and
2. A software that will allow comparisons between the existing methods and the proposed ones. This is resolved by the development of ABS' Eagle wind software.

ABS Eagle Wind

In Support of the SNAME OC-1, ABS has developed the software algorithms that integrates the Fluid Drag Quadratic Equation over a geometrically defined plane surface ABS Eagle Wind allows the calculation of wind forces and moments for an offshore asset defined by planar polygons, cylinders and truncated cones. The calculations may be in strict compliance with the original rules and regulations, or the Fluid Drag Quadratic Equation combined with the power-law boundary layer profile.

The software, while originally devised for afloat conditions, allows the user to position the model afloat in any combination of draft-heel-trim and relative wind direction. This set of parameters with the correct modeling, can also be used to determine wind forces and moments on jack-up units in the elevated condition.

Figure 2 – ABS Eagle wind model of a jack-up on its development screen

The software also allows the selection of air density (ρ), velocity reference height (Zr), and the exponent of the power curve (α). These elements can be defined by their boundary polygons, and be affected by permeability/shielding, and the effect of the angle of wind incidence can be considered individually.

Integration of the Boundary Layer profile and the geometrical model.

Following the development of the equations and the integration of the continuous wind pressure function over an arbitrary area is a mathematically correct approach for determining wind force and wind moment. The closed-form solution for each area exposed to wind pressure provides an advantage over other discrete, block techniques due to the improvement in computation time. The correctness of the equations set forth in the Appendix have been tested with simple geometrical forms compared to results from Eagle Wind from the same models.

Conclusions

The implementation of the new empirical methods discussed above to evaluate wind forces and moments for design, classification and regulatory approval represents an advancement for the offshore industry. Such methods are an important part of the design spiral and can provide results that are benchmarked to those that have been proven and accepted for many years. The advancement comes with methods that can be used on modern high-speed computers, generating results efficiently and consistently for all users. Work remains to be done to implement the tools presented in this paper into wind load software and then have them vetted and accepted by all regulatory agencies.

Acknowledgements

The Authors would like to acknowledge the uninterrupted support of the Society of Naval Architects and Marine Engineers, the members of the SNAME T&R Committee, the members of the SNAME Offshore Committee, and the many industry volunteers participating in SNAME OC-1 panel.

Special thanks must also be given to the ABS Technology Department, for its financial support of this working group and for the development of the ABS Eagle Wind software, and for releasing the same for the use of the industry at large.

---

[1] US Code of regulations, Title 46, Shipping – Chapter S - §174.055 Calculation of wind heeling moment (Hm)

[2] Technical & Research Bulletin 5-4 – "Guidelines for Wind· Tunnel Testing of Mobile Offshore Drilling Units" - Panel OC-1 (Stability And Motions Technology) of the Offshore Committee; 1988

[3] American Bureau of Shipping – Rules for the Building and Classing Offshore Mobile Drilling Units, 1968

[4] Code for the Construction and Equipment of Mobile Offshore Drilling Units, 2009 (2009 MODU Code), 2010

[5] US Coast Guard, Merchant Maritime Technical Note 6-65, 1965

[6] SNAME - Guidelines for Site Specific Assessment of Mobile Jack-Up Units T&R Bulletin 5-5 and 5-5A, PANEL OC-7 Site Assessment of Jack-Up Rigs, 2008

[7] Wikipedia – Density of Air, 30 April 2015

APPENDIX

Wind Forces and Moments

Integration of the Boundary Layer profile and the geometrical model.

Further to establishing a profile for the boundary layer, combining the continuous wind pressure function over an arbitrary area is a means for determining wind force, and wind moment. The closed-form solution for each area exposed to wind pressure provides an advantage over other discrete, block techniques due to the improvement in computation time.

The Fluid Drag Quadratic Equation

$$F = \frac{1}{2}\rho v_{(z)}^2 C_d A$$ Where; [1]

F: is the wind force or fluid drag force in N, Kg_f, or Lb_f (subscript f indicates force units)

ρ: is the density of air – 1.222 Kg_m/m^3 (subscript indicates mass units) used by the rules and regulations and 1.225 Kg_m/m3, 0.0023769 slug/ft^3, or 0.0765 Lb_m/ft^3) according to ISA (International Standard Atmosphere)8.

A: is the area of the body projected on a surface perpendicular to the direction of wind.

v(z): is the velocity of the fluid corrected for the increase of velocity with the rise in elevation.

Cd: is the drag coefficient – a non-dimensional coefficient established on the basis of wind tunnel measurements

Ch: is the height coefficient that accounts for the boundary layer effect

The Boundary Layer Profile

The power-law Boundary Layer Profile

$$\frac{v(z)}{V_r} = \left[\frac{z}{Z_r}\right]^{\propto}$$ Where; [2]

V(z) is the wind velocity at an elevation z above the sea level

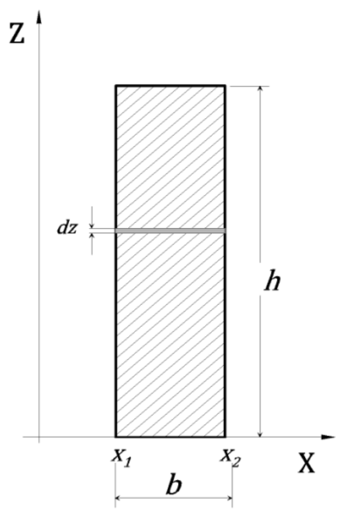
Vr is the nominal velocity established for the design (typically 100 knots)

Zr is the nominal elevation at which the wind velocity *Vr* is established.

Thus;

$$v(z) = V_r \cdot \left[\frac{z}{z_r}\right]^{\alpha}$$ [3]

Forces and moments on a simple rectangle



Assume a rectangle of height h, width b, and with its base on z=0

$$dF = \frac{1}{2}\rho Cs \cdot \frac{V_r^2}{z_r^{2\alpha}} \cdot b \cdot z^{2\alpha} dz$$ Where

$$C_1 = \frac{1}{2}\rho Cs \frac{V_r^2}{z_r^{2\alpha}} \cdot b$$

$$F = C_1 \int_{z_1}^{z_2} z^{2\alpha} dz = \frac{C_1}{2\alpha+1} \cdot z^{2\propto+1}\Big]_{z_1}^{z_2}$$ [4]

In the case of the simple rectangle $Z_1 = 0$ and $Z_2 = h$ we have

$$F == \frac{C_1}{2\alpha + 1} \cdot h^{2\propto+1}$$

$$dM = c_1 z^{1+2\alpha} dA = c_1 \cdot z^{1+2\alpha} \cdot b \cdot dz$$

$$M = C_1 \int_{z_1}^{z_2} z^{2\alpha} \cdot z \cdot dz = \frac{C_1}{2\alpha+2} \cdot z^{2\alpha+2} \Big]_{z_1}^{z_2} \qquad [5]$$

Again, for $Z_1 = 0$ and $Z_2 = h$ we have

$$M = \frac{C_1}{2\alpha + 2} \cdot h^{2\alpha+2}$$

And the elevation above the base line of center of pressure *Cp* is

$$Cp = \frac{M}{F} \qquad [6]$$

For example we will apply the above to a rectangle 100m high and 1m wide

Vr = 51.4444 m/s
Zr = 10m
ρ = 1.222 kg/m^3
h = 100m
b = 1m
Cs = 1
g = 9.80665 m/s^2

Applying equation [4]

$$C_1 = \frac{1}{2} \cdot 1.222 \cdot 1 \cdot \frac{51.4444^2}{10^{0.2}} = 1020.275$$

$$F = \frac{C_1}{2 \cdot 0.1 + 1} \cdot 100^{1.2} = 213{,}568 \text{ N} = 21778 \text{ Kg}_f$$

Applying equation [5]

$$M = \frac{C_1}{2\alpha + 2} z^{2\alpha+2} = \frac{1020275}{2.2} \cdot 100^{2.2} = 11{,}649{,}163\ N = 1{,}187{,}884\ Kg_f m$$

And the elevation of center of pressure *Cp* above the base line is

$$Cp = \frac{M}{F} = \frac{1{,}187{,}884\ Kgm}{21{,}778\ Kg} = 54.545\ m$$

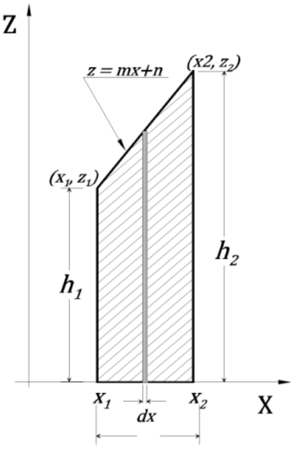
Forces and moments on a Trapezoid

Assume a trapezoid of height h_1 and h_2, width b, and with its base on z=0

The upper boundary of the trapezoid is defined by the points with coordinates (x_1, z_1) and (x_2, z_2) and the line defined by the two points may be expressed as;

$z = mx + n$ Where;

$$m = \frac{z_2 - z_1}{x_2 - x_1} \quad and \quad n = z_1 - x_1 \cdot \frac{z_2 - z_1}{x_2 - x_1}$$



Applying equation [4]

$$dF = \frac{C_1}{2\alpha + 1} z^{2\alpha+1} dx$$

$$dF = \frac{C_1}{2\alpha + 1} (mx + n)^{2\alpha+1} dx$$

$$F = \frac{C_1}{2\alpha + 1} \int_{x_1}^{x_2} (mx + n)^{2\alpha+1} dx = \frac{C_1}{2\alpha + 1} \cdot \frac{1}{m(2\alpha + 2)} (mx + n)^{2\alpha+2} \Big]_{x_1}^{x_2}$$

Given that; $mx_1 + n = z_1$ and $and\ mx_2 + n = z_2$

We can express the equation as:

$$F = \frac{C_1}{2\alpha + 1} \cdot \frac{1}{m(2\alpha + 2)} (z_2^{2\alpha+2} - z_1^{2\alpha+2})$$

And since

$z_1 = h_1$ and $z_2 = h_2$

We can also write the expression as

$$F = \frac{C_1}{2\alpha+1} \cdot \frac{1}{m(2\alpha+2)} (h_2^{2\alpha+2} - h_1^{2\alpha+2}) \qquad [7]$$

Applying equation [5]

$$dM = \frac{C_1}{2\alpha + 2} \cdot z^{2\alpha+2} dx = \frac{C_1}{2\alpha + 2} \cdot (mx + n)^{2\alpha+2} dx$$

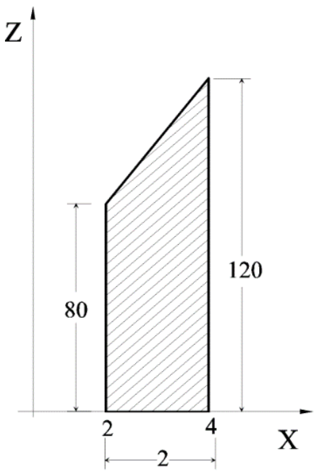
$$M = \frac{C_1}{2\alpha + 2} \cdot \int_{x_1}^{x_2} (mx + n)^{2\alpha+2} dx = \frac{C_1}{2\alpha + 2} \cdot \frac{(mx + n)^{2\alpha+3}}{m(2\alpha + 3)} \bigg]_{x_1}^{x_2}$$

$$M = \frac{C_1}{2\alpha+2} \cdot \frac{1}{m(2\alpha+3)} \cdot ({z_2}^{2\alpha+3} - {z_1}^{2\alpha+3}) \qquad [8]$$

And the elevation of center of pressure Cp above the base line is

$$Cp = \frac{M}{F}$$

As an example we will apply to a trapezoid 2m wide, with its base on z=0 and 80m and 120m height on each end respectively.



$V_r = 51.4444$ m/s
$Z_r = 10$m
$\alpha = 0.1$
$\rho = 1.222$ kg/m^3
$h_1 = 80$
$h_2 = 120$
$b = 2$m
$C_s = 1$
$g = 9.80665$ m/s^2

$$C_1 = \frac{1}{2} \cdot 1.222 \ \cdot 1 \cdot \frac{51.4444^2}{10^{0.2}} = 1020.275$$

Determine the coefficients of the line defined by the points (2, 80) and 4, 120)

$$z = mx + n$$

$$m = \frac{120 - 80}{4 - 2} = 20 \quad and \quad n = 80 - 2 \cdot \frac{120 - 80}{4 - 2} = 20$$

Applying equation [7]

$$F = \frac{C_1}{2 \cdot 0.1 + 1} \cdot \frac{1}{20 \cdot (2 + 0.2)} \cdot \quad (120^{2.2} - 80^{2.2}) = 427{,}821 \text{ N} = 43{,}626 \text{ Kg}_f$$

Applying equation [8]

$$M = \frac{C_1}{2\alpha + 2} \cdot \frac{1}{20(2\alpha + 3)} \cdot (120^{3.2} - 80^{3.2}) = 23{,}708{,}246 \ Nm$$

$$= \ 2{,}417{,}568.3 \ Kg_f m$$

$$Cp = \frac{M}{F} = \frac{2{,}417{,}593\ Kgm}{43{,}626 Kg} = 55.416\text{m}$$

Wind Forces and Moments on an irregular Polygon

Assume a polygon such as the one on the illustration. The polygon may be resolved with a sequential sum of the properties of the trapezoids formed by consecutive points.

Applying equation [7]

$$F = \sum_{i=1}^{n} \frac{C_1}{2\alpha + 1} \cdot \frac{1}{m_{i-(i-1)}(2\alpha + 2)} \left(z_i^{2\alpha+2} - z_{i-1}^{2\alpha+2}\right) \qquad [9]$$

Where;

$$m_{i-(i-1)} = \frac{z_i - z_{i-1}}{x_i - x_{i-1}}$$

Applying equation [8] [10]

$$M = \sum_{i=1}^{n} \frac{C_1}{2\alpha + 2} \cdot \frac{1}{m_{i-(i-1)} \cdot (2\alpha + 3)} \cdot \left({z_i}^{2\alpha+3} - {z_{i-1}}^{2\alpha+3}\right)$$

And applying equation [6]

$$Cp = \frac{M}{F} \qquad [6]$$

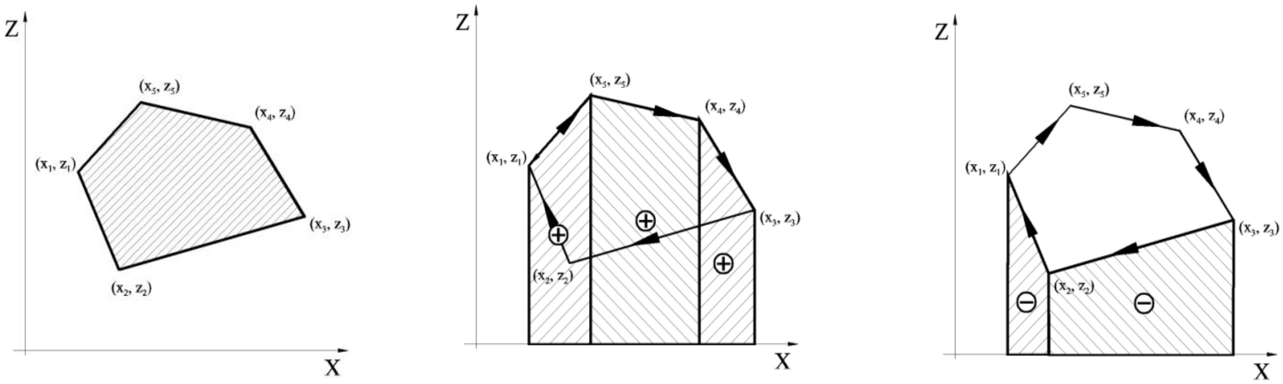


Figure 3 - Integrating a polygon by way of sequential trapezoids

8 Wikipedia – Density of Air, 30 April 2015