EFFECTIVE SHEAR AREAS OF JACK-UP CHORDS – DEVELOPMENT OF A LOGICAL APPROACH TO ASSESSMENT

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ABSTRACT

This paper presents the background to, and the results of, analyses carried out by Brad Mobbs[1] on the effective shear areas of jack-up leg chords [Ref 1 & 2]. These analyses, using a combination of finite element analysis (FEA) and traditional shear area calculations, were carried out to help validate proposed changes to the ISO Draft International Standard (DIS) "ISO DIS 19905-1 Site-specific assessment of mobile offshore units – Part 1: Jack-ups" that was issued by ISO on 9th September 2009. The proposed changes were to include some example chord configurations and give associated sum-of-the-parts shear area calculations. However, the panel developing the examples wanted some validation of their acceptability.

The results of the FEAs validated the more refined sum-of-the-parts approaches to a limited extent, but by indicating how the shear stress was distributed across the different cross-sections, they also identified areas where changes should be made to improve the accuracy of the approach. The proposed example calculations were then modified to suit the validation.

A limited number of errata have been incorporated into second edition of ISO 19905-1 (published 15th January 2016) [Ref 3]. It is planned that the example diagrams and the rest of the changes will be incorporated into the third edition that is due to be published in 2019/2020.

This paper expands on the background to the study, details the methodology used in the finite element analyses, presents the results of the FEA, and discusses the conclusions.

The body of this paper sets out only a limited sub-set of the results of the calculations. The full reports, as submitted to the ISO panel in charge of this aspect of ISO 19905-1, will be found in ISO TR 19905-2 second edition [Ref 4] when it is published in 2019/2020.

BACKGROUND

During the development of the jack-up site assessment ISO standard it was realized that the proposed final document would need to be tried-out and benchmarked against the existing assessment methodology (SNAME Bulletin 5-5A [Ref 5]). A number of consultants were chosen to carry out this work, the results of which are documented in OTC 22949 [Ref 6]. This work started in November 2009. At the same time, one consultant, Noble Denton, was instructed to produce a "go-by" document that set out, in detail, an example calculation. The purpose of this go-by was to help analysts use the prospective ISO 19905-1. Many questions were raised by the consultants carrying out the benchmarking work and during the go-by document's development: one that has been subject to much discussion relates to the shear area to be used when calculating the shear strength of jack-up leg chords.

The DIS copy of 19905-1, used in the development of the go-by document, contained only a simple table giving suggested values for the shear areas of component part (see Table 1). The document suggested that these component part values could then be "summed" to calculate the effective shear area of fabricated leg

[1] The study was started when Brad Mobbs worked for Cameron Drilling Systems (CDS) who had acquired the company he originally worked for, LeTourneau Inc. The jack-up design group within CDS was later spun off and merged with Keppel resulting in Keppel LeTourneau USA. Brad Mobbs carried out the analyses described herein. John Stiff is the panel convenor for 19905-1 Acceptance Criteria (IOGP ISO TC67/SC7/WG7 Panel 10).

chords. While this represented a potentially workable solution, there was little detailed advice on how to actually carry out the sum for a particular leg chord. A methodology could be inferred for "teardrop" chords, but significant interpretation would be required for a chord containing components of circular cross-section.

An attempt was made to correct this omission by formalizing two simple "sum-of-the-parts" approaches. Diagrams of generic chord types were developed, and the effective shear area calculation of each were detailed within a table. These suggested methodologies were considered by, and subject to considerable debate within, the relevant ISO Panel, but there was a strong desire for them to be calibrated against more detailed analysis. To achieve this, one of the authors volunteered to undertake some finite element analyses. A parametric array of cross sections was developed and analyzed to determine reasonable effective shear areas that could be used in future analyses.

Compounding the problem with the DIS was that the table of shear properties contained inconsistent suggested values. For example, the suggested shear area for a flat plate (parallel to a long edge) was 90% of the plate cross sectional area. This is reasonable for determining the plastic limits. Conversely, the value for a tube was given as 50% of the cross sectional area: correct for an elastic analysis, but not the plastic limit used in the rest of the standard. The table contained some other limits (e.g. angle between a plate and the shear direction) that caused inconsistent, and in some cases ludicrous results (see discussion in section on teardrop chords below).

Section	Effective shear area, A_v
Rolled I, H and channel sections, load parallel to web	$t\,D_s$
Welded I sections, load parallel to web	$t\,d$
Rectangular hollow sections, load parallel to webs	$A\,D_s/(D_s + B_s)$
Welded box sections, load parallel to web	$2\,t\,d$
Rolled Tee-sections, load parallel to web	$t\,D_s$
Welded Tee-sections, load parallel to web	$t\,(D_s - T)$
Circular hollow sections	$0{,}5\,A$
Solid bars and plates	$0{,}9\,A$
Closed sections with inclined plates	$0{,}9\,\Sigma[\cos(\theta_i)\,A_{oi}]$

T	is the flange thickness of a welded T-section.
t	is the web thickness.
D_s	is the overall depth of cross-section.
d	is the web depth; for rolled sections measured with respect to root radii, for welded sections measured between inside faces of flanges.
B_s	is the overall breadth of cross-section.
A	is the area of cross-section.
A_{oi}	is the area of rectilinear component *i*.
θ_i	is the angle between *the shear force* direction being considered and the larger dimension of the cross-section of component *i* for $\theta_i \leq 45°$. $\cos(\theta_i) = 0$ if $\theta_i > 45°$

Table 1 Shear Properties Table in 19905-1 prior to Current Study

BASIC REQUIREMENT FOR SHEAR AREA CALCULATIONS

It is important to note at the outset that shear stresses are rarely the limiting factor when undertaking a site-specific assessment of a jack-up – chord shear stresses should be checked, but the expectation is that they will not be critical. This understanding helps drive how the methodology for shear stress assessment is set out in the standard: a simple method that is normally conservative is proposed, but more complex and accurate calculations are permitted.

The basis for the member strength calculation in ISO 19905-1 is ultimate limit state (ULS), so member strength calculations are based on the full plastic capacity. The plastic capacity for members subject to axial and bending loads are relatively well established (although combined loadings are more complex), but it is more difficult to establish the limiting shear load capacity. It can be done, as evidenced by the work presented in this paper, but given that shear is so rarely the limiting factor, how worthwhile is it to expend significant time and energy on something that will normally not be critical? Hence the requirement for a simple, but conservative, check.

VALIDATION OF APPROACH USING SIMPLE TUBE

The first step in assessing the relatively complex chord shapes was to demonstrate that the proposed FEA would produce expected results for a simple tubular cross section. It immediately became apparent that the loading and support conditions to obtain pure shear at a cross section was not trivial. A number of approaches were tried, and not all the analyses used the same approach, but validation of the results across the different support conditions was demonstrated. The final, and best loading/support arrangement is shown in Figure 1, but even with this set of boundary conditions, it was necessary to establish that the model was sufficiently long to have limited influence on the results.

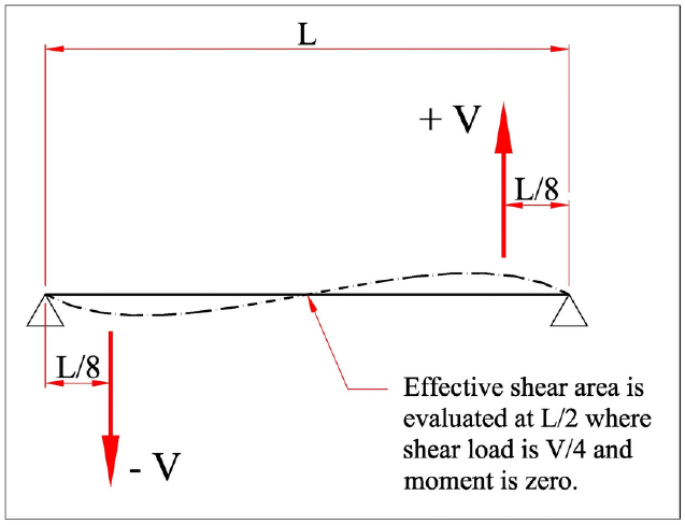


Figure 1 Loading and Support Conditions to achieve Shear Only Loading at Assessment Cross Section

The calibration analyses used a tube with outside diameter of 12.75 inches and a wall thickness of ¾ inch giving a cross sectional area of 28.27 square inches. Material yield strength, Young's (elastic) modulus, and Tangent (plastic) modulus were specified to be 100 ksi, 30,000 ksi, and 600 ksi (2% of Young's Modulus), respectively. The resulting limiting shear area factor was 0.62 which compares well to the theoretical value of 0.637 (2/π). Note that, as used in this paper, the Shear Area Factor is defined as:

$$Shear\ Area\ Factor = \frac{Shear\ Load\ at\ critical\ cross\ section}{(Average\ Shear\ Stress\ at\ \ critical\ shear\ section)\ x\ (cross\ section\ area)}$$

Figure 2 shows the shear stress distribution for the load level at which the analysis failed to converge, and a plot of the effective shear area factor vs. shear load on the cross section. As can be seen, the maximum shear stress is at the neutral axis of the tube, and the effective shear stress factor becomes asymptotic to just over 0.62.

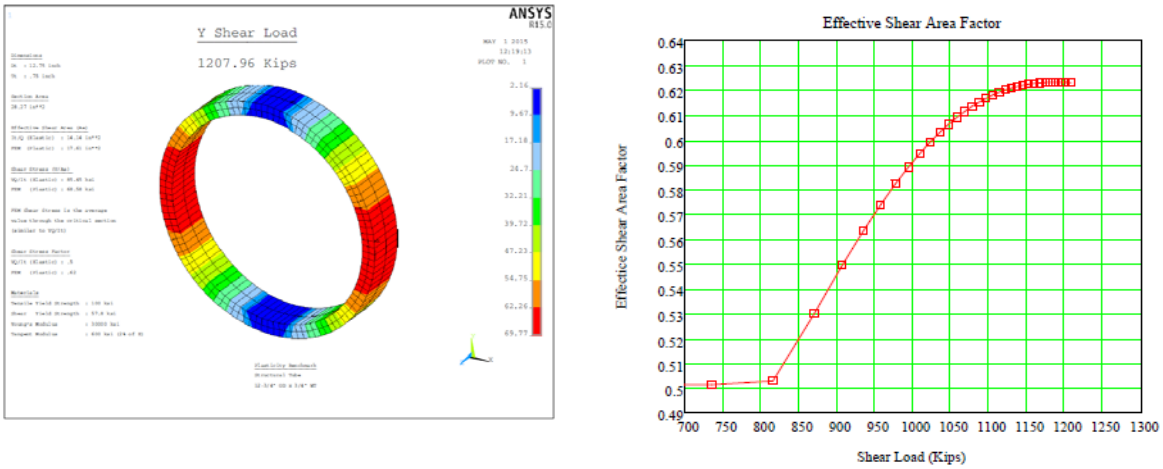


Figure 2 FEM of Calibration Tubular and Effective Shear Area Factor vs. Y Shear Load

At a later date the tubular analysis was repeated, but for tubes of different D/t ratios. Initially this additional study used 2D plastic "beam" elements to check their suitability, but the final results were based on analyses using 3D plastic "solid" elements. It is interesting to note that in order to generically derive an effective shear area close to the theoretical value of 0.637, it was necessary to reduce the tangent modulus to 0.1% of Young's modulus. It also transpired that the results from the 3D model were far more sensitive to the tangent modulus than were the 2D models.

The validity of the analyses was checked over a range of D/t ratios, and found acceptable over the full range checked: from D/t of 20 to 60.

TUBULAR BASED CHORDS

Having validated the model, the next step was to assess some of the example models contained in the proposed changes to 19905-1. Figure 3 is extracted from the draft 3rd edition of 19905-1 and shows some of the proposed chord shape examples containing tubular sections. (Note that this is a later copy of the diagrams than was used for the ISO assessment in the study.) Cross section (a) and (b) are similar, except note that in case (b) the wall thickness of the tubular component thins as it joins the rack. This is important as the shear stress is maximized close to the neutral axis, when the applied shear is in the "Y" direction, and at this point the wall thickness is minimized (see also Figure 4). It was a concern to the panel that the shear stress at this section could be underestimated by a simple sum-of-the-parts assessment. As it transpired some changes were made to the proposed calculations that overcame the problem.

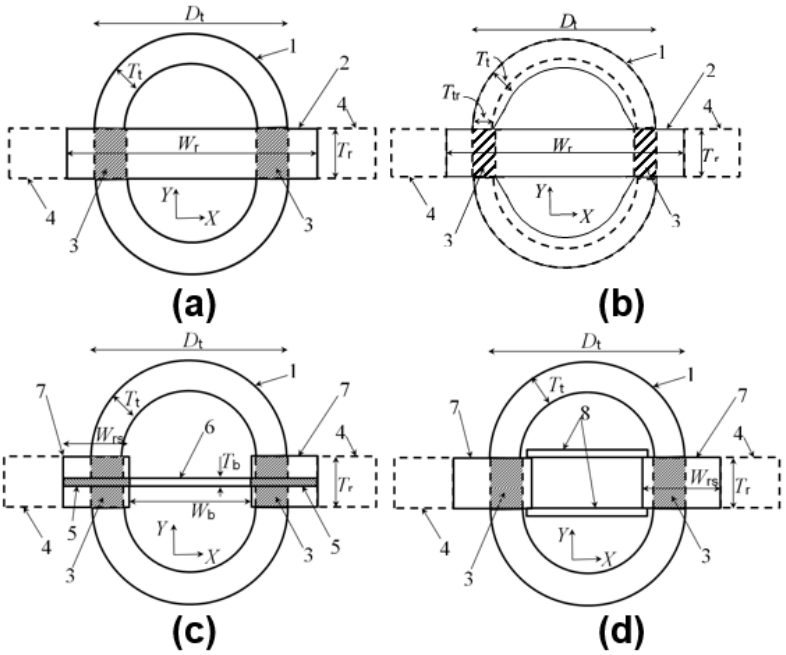


Figure 3 Example Chord cross sections that contain Tubular Components

The FEAs assessed eight parametric cases, two of type (a) and six of type (b) (see Figure 3). (This is logical as most tubular chords of this type are closer to case (b) than to case (a).) Shear loads were applied in both the "X" and "Y" directions. Calculations were carried out using the draft version of 19905-1 available at the time, a version with some additional proposed changes, standard elastic shear theory, and finite element analysis.

The basis for the standard elastic theory is the formula for average shear stress on the shear section being assessed:

$$Shear\ Stress\ \sigma_v = VQ/It$$

Where:

- V = Shear load (in the X or Y direction) on the cross section
- Q = 1st moment of area, about the neutral axis, between the shear section being assessed and the outer fiber
- I = 2nd moment of area of the entire cross section
- t = The combined width of material at the shear section being assessed perpendicular to the shear load

Figure 4 shows the layout of an example cross section along with the axes and the critical shear sections (axes in blue) assessed using the standard elastic model. It can be seen, as discussed above, that the critical shear section for a shear load applied in the "Y" direction is just above the rack where the tubular component is thinnest in case (b).

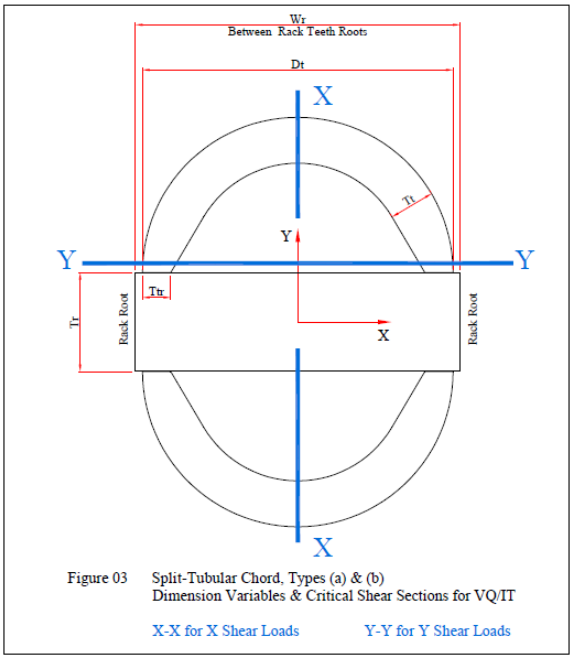


Figure 4 Example of Tubular Chord Cross Section showing axes and location of critical shear stress checks

The draft version of 19905-1 which was used to develop the effective shear area for the (b) cases, utilized 50% of the total tubular area as its contribution to effective shear area for Y shear loads (see "Current ISO" cases in Table 2). Based upon the results of this study, the tubular contribution to effective shear area for Y shear loads was modified to 60% of the total tubular area calculated using a constant wall thickness which equals the wall thickness at the point where it attaches to the rack. This revision was used for the "ISO Revised" cases in Table 2. (Note that the connection between tubular and rack must be a full penetration weld, and the wall thickness where the tubular attaches to the rack cannot be taken to be greater than the wall thickness for the body of the tubular – Ttr $\leq$.Tt in Figure 4.)

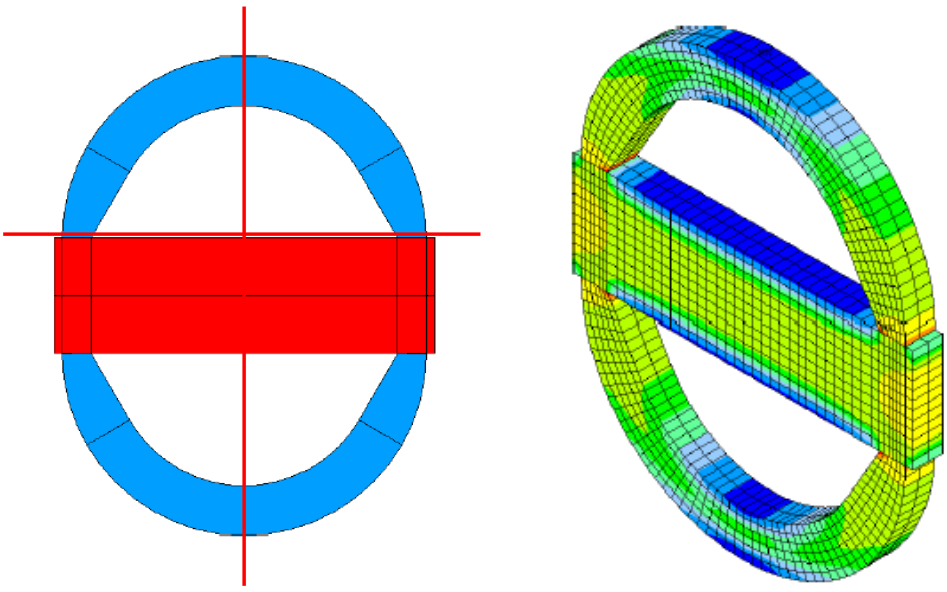


Figure 5 Example of Split Tubular Chord type (b) and FEA output showing highest shear stress at rack/tubular interface due to "Y" Shear Load

Figure 5 shows one of the tubular cross sections assessed. It can be seen in the stress plot on the right that the highest shear stress is at the interface between the edge of the split tubular and the rack wall. This is where elastic analysis would suggest the highest shear stress occurs and is confirmed by the FEA. It is also of note that it appears that the entire width of the rack between the opposed rack tooth roots is effective in carrying the "Y" shear. The "Current ISO", as referenced in Table 2, used only a small amount of the rack to resist shear in the "Y" direction, as shown by the two shaded areas on the rack in Figure 3. It was realized that this was unreasonably conservative, based on the FEA analyses, so the rack's contribution to the effective "Y" shear area was increased to 90% of its full area (0.9 x Wr x Tr) in the "Revised ISO" and in the likely 3rd edition.

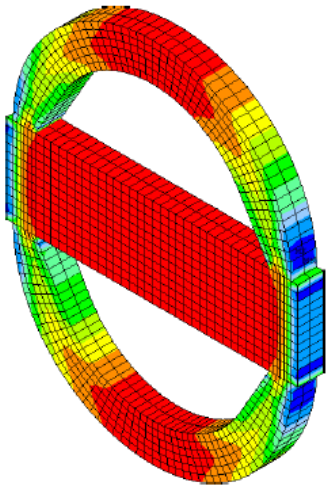


Figure 6 Split Tubular Chord type (b) FEA output showing shear stress distribution due to "X" Shear Load

Figure 6 shows the same split tubular chord shown in Figure 5, but under "X" direction shear load. It is interesting to note that the outstand parts of the rack that extend beyond the split tube are carrying little load (as evidenced by their blue color). The "Current ISO" results presented in Table 2 are based upon 90% of the rack area between the rack tooth roots as being effective for X shear loads. Based upon the results of these analyses, the rack area contribution for X shear loads was modified to 100% of the rack area associated with the tubular diameter Dt ("Revised ISO" in Table 2). A factor of 0.9 was later added to this so the current 3rd edition draft has the rack area contribution as 0.9 x Dt x Tr.

The results of the analyses are presented in Table 2. The top part of the table gives the parametric dimensions for the eight split tubular chords modelled and assessed in the study. The dimension descriptions are given, but also align with the dimensioning abbreviations in Figure 3.

Table 2 gives the results for shear in the "X" direction (parallel to the rack) and the "Y" direction (perpendicular to the rack). The most informative numbers are values for the ratio of the effective shear area to the total cross sectional area for the "Revised ISO" and the "FEA" (the bottom two rows of the "X" shear cases and the "Y" shear cases) labelled as Ae/A. It can be seen that for "X" shear the FEA effective shear area ratio is always a little larger than the revised ISO ratio. This demonstrates that the revised ISO method is always conservative for the cases studies. (It is of note that the current 3rd edition introduction of an additional 90% factor on the rack area will increase this conservatism.)

Looking at the same effective shear area ratio for the "Y" direction loading shows that in all but one case the revised ISO is more conservative than the FEA results. In the one case where the FEA ratio is lower than the revised ISO case (case 8 in the last column) the difference is very small. In addition, this is an

unrealistic case of a thin split tube on an extremely thick rack (see Figure 7). It was decided that this un-conservatism was so small, and the case so unlikely, that a change to the draft 3rd edition was not warranted.

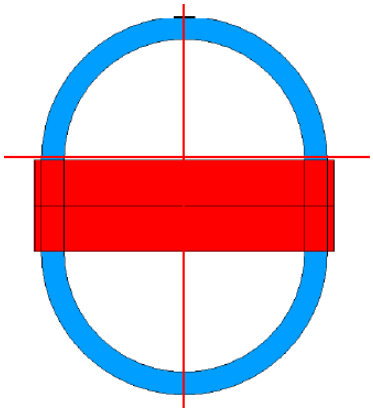


Figure 7 Case 8 in which the "Revised ISO" calculated effective shear area was marginally larger than that derived from the FEA

At one stage some other split tubular configurations had been considered for inclusion in the draft 3rd edition of 19905-1 (e.g. chords that have only one semicircular section) but it was decided that these were too complicated and uncommon to warrant the additional work. The analyst is allowed to develop their own rational methodology, and conservative assumptions could likely be developed without too much effort.

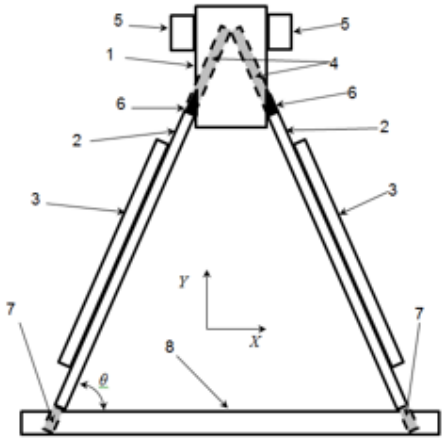
Table 03				Split-Tubular Chords Type (a) or (b) from Proposed ISO Figures							
				01	02	03	04	05	06	07	08
				Type (b)						Type (a)	
Rack Width	(between teeth)	Wr	Inch	23.00	23.00	**16.00**	23.00	23.00	23.00	23.00	23.00
Rack Thickness		Tr	Inch	7.00	**4.00**	7.00	7.00	7.00	7.00	7.00	7.00
Tubular Diameter		Dt	Inch	22.00	22.00	**15.00**	22.00	22.00	22.00	22.00	22.00
Tubular Thickness		Tt	Inch	3.00	3.00	3.00	**5.00**	3.00	3.00	3.00	**1.75**
Tubular Thickness @ Rack Connection		Ttr	Inch	1.75	1.75	1.75	1.75	**1.00**	**2.50**	**3.00**	1.75
				SECTION AREAS							
Rack Area	(between teeth)	Ar	Inch**2	161.00	92.00	112.00	161.00	161.00	161.00	161.00	161.0
Tubular Areas	(including welds)	At	Inch**2	172.09	172.09	108.10	244.83	165.44	177.24	179.07	111.33
Total Area			Inch**2	333.09	264.09	220.10	405.83	326.44	338.24	340.07	272.33
	X SHEAR LOAD **Parallel to Rack**			**It/Q & FEA are at x=0** **(y-axis section at neutral axis)**			**Current ISO** **Revised ISO**	**Ae = (0.9 * Wr * Tr) + (0.5 * At)** **Ae = (1.0 * Dt * Tr) + (0.6 * At)**			
It /Q	Elastic	Effective Shear Area		Inch**2	197.66	151.72	133.36	248.57	196.80	198.08	198.17	163.98
ISO	Current	Effective Shear Area	Ae	Inch**2	230.95	168.85	154.85	267.32	227.62	233.52	234.44	200.57
ISO	**Revised**	**Effective Shear Area**		**Inch**2**	**257.25**	**191.25**	**169.86**	**300.90**	**253.26**	**260.34**	**261.44**	**220.80**
FEA	**Plastic**	**Effective Shear Area**		**Inch**2**	**286.69**	**222.03**	**190.01**	**363.57**	**281.66**	**291.58**	**293.98**	**230.28**
It /Q	Elastic	Shear Area Factor			0.59	0.57	0.61	0.61	0.60	0.59	0.58	0.60
ISO	Current	Shear Area Factor	Ae/A		0.69	0.64	0.70	0.66	0.70	0.69	0.69	0.74
ISO	**Revised**	**Effective Shear Area**			**0.77**	**0.72**	**0.77**	**0.74**	**0.78**	**0.77**	**0.77**	**0.81**
FEA	**Plastic**	**Shear Area Factor**			**0.86**	**0.84**	**0.86**	**0.90**	**0.86**	**0.86**	**0.86**	**0.85**
	Y SHEAR LOAD **Perpendicular to Rack**			**It/Q & FEA are at Tr / 2** **(x-axis section at edge of rack)**			**Current ISO** **Revised ISO**	**Ae = (0.5 * At) + (0.9 * Tr * Ttr * 2.0)** **Ae = (0.6 * Atr) + (0.9 * Wr * Tr)**			
It /Q	Elastic	Effective Shear Area		Inch**2	77.29	65.74	60.09	73.44	44.64	109.55	131.10	80.83
ISO	Current	Effective Shear Area	Ae	Inch**2	108.10	98.65	76.10	144.47	95.32	120.12	127.34	77.72
ISO	**Revised**	**Effective Shear Area**		**Inch**2**	**211.70**	**149.60**	**144.51**	**211.70**	**184.48**	**236.79**	**252.34**	**211.70**
FEA	**Plastic**	**Effective Shear Area**		**Inch*2**	**232.68**	**172.32**	**157.32**	**294.07**	**206.19**	**250.59**	**261.27**	**191.97**
It /Q	Elastic	Shear Area Factor			0.23	0.25	0.27	0.18	0.14	0.32	0.39	0.30
ISO	Current	Shear Area Factor	Ae/A		0.32	0.37	0.35	0.36	0.29	0.36	0.37	0.29
ISO	**Revised**	**Effective Shear Area**			**0.64**	**0.57**	**0.66**	**0.52**	**0.57**	**0.70**	**0.74**	**0.78**
FEA	**Plastic**	**Shear Area Factor**			**0.70**	**0.65**	**0.71**	**0.73**	**0.63**	**0.74**	**0.77**	**0.71**

2.0 correction factor was added to Current ISO formulae for Y shear load. See 3-1 for explanation.
Atr is the total split-tubular area associated with a constant wall thickness of Ttr and diameter Dt. $\pi * [(Dt*Ttr) - Ttr^2]$ for two 180° segments

Table 2 Results of the Analyses of the various Split Tube Chords using different methods

TEARDROP CHORDS

When the work described in this paper was started, the draft ISO contained a proposed method to calculate the effective shear area for "teardrop" type chords. The method entailed creating artificial areas within the rack and back plate to account for material that was thought likely to be effective, but without significant justification. The shear areas presented in Figure 8 are an example of one of the proposed approaches. However, it can be seen that the shear areas of the side plates depend on the angle of the plate to the applied load, and if that angle is greater than 45°, then the side plate is considered ineffective. This resulted in some (artificial) teardrop chord shapes having a zero effective shear area. As discussed above, it was realized that this anomaly needed to be corrected, and it has been, but the difficulty in justifying the calculations for a teardrop chord seemed fruitless given that the conventional elastic shear calculations would normally prove to be adequate. Based on this, the report and this paper suggest that the most expedient approach for teardrop chords is to use the It/Q elastic method given above.



Key

1	Structural part of chord rack, X direction width W_r and Y direction length L_r
2	Chord side plate with an area of cross section of A_s
3	Chord side plate reinforcing plate
4	Two artificial areas representing extension of chord side plates into the rack, each with an area totally within the rack and with no overlap of A_{sr}. A_{sr} =0 if weld item 6 is not effectively full penetration
5	Chord rack reinforcing plates
6	Weld between the chord side plates and chord racks.
7	Two artificial areas representing extension of chord side plates into the chord back plate, each with an area totally within the chord back plate of A_{sb}. A_{sb}=0 if weld between the chord side plate and chord back plate is not effectively full penetration.
8	Chord back plate with area of cross section of A_b
θ	Angle between the chord back plate and the chord side plate in degrees
X	Axis parallel to the chord back plate
Y	Axis perpendicular to the chord back plate
A_{vx}	Shear area in the X direction. The value of A_{vx} can be determined from rational analysis. In lieu of such analysis, the following values may be used: $A_{vx} = 0{,}9\ [2(A_s + A_{sr})\cos\theta + A_b]$ for $\theta \le 45°$. $\cos\theta = 0$ if $\theta > 45°$
A_{vy}	Shear area in the Y direction. The value of A_{vy} can be determined from rational analysis. In lieu of such analysis, the following values may be used: $A_{vy} = 0{,}9[2(A_s + A_{sr} + A_{sb})\cos(90° - \theta)]$ for $90° - \theta \le 45°$. $\cos\theta = 0$ if $(90° - \theta) > 45°$

Figure 8 Early proposal for sum-of-the-parts effective shear area calculation for teardrop chords

CONCLUSIONS

The shear stress assessment clause in 2nd edition of ISO 19905-1 needs to be revised to include some example effective shear area calculations. Some sum-of-the-parts examples were developed and discussed in the relevant ISO panel. One of the authors (Brad Mobbs) volunteered to run some FEA to check the veracity of the split tubular examples, and to assess the teardrop chord example.

The results of the analyses verified that the proposed approach was suitable for the split tubular chords, albeit with some modifications.

During the analyses it became apparent that the base table for calculating the effective shear areas of other shapes contained some inconsistencies. These have been corrected and will be included in the 3rd edition of 19905-1.

The assessment suggested that the proposed sum-of-the-parts approach was not suitable for teardrop chords and that they could be (conservatively) assessed using conventional elastic theory (accepting that ISO 19905-1 is based on plastic methods).

3rd Edition of 19905-1, to be published in 2019/2020, will be updated to include the results of this study.

ABBREVIATIONS

DIS	Draft International Standard
FDIS	Final Draft International Standard
FEA	Finite Element Analysis
FEM	Finite Element Model
ISO	International Organization for Standardization
ksi	Kips per square inch (the FEA were carried out in, and the results presented in, imperial units)
SNAME	Society of Naval Architects and Marine Engineers
TR	Technical Report (associated with an ISO standard, as in ISO TR 19905-2)
ULS	Ultimate Limit State

D	Diameter (of a tubular member)
Dt	Diameter of the split tubular component of a leg chord
I	Second moment of area (of a member)
Q	First moment of the area, about the neutral axis, between the section being assessed for shear and the outer fiber
t	Wall thickness (of a tubular member)
Tr	Thickness of the rack component of a chord
Tt	Wall thickness of the split tubular component of the chord with diameter Dt
Ttr	Wall thickness of the split tubular component where it is welded to the rack
Wr	Width of the rack

REFERENCES

[1] Brad Mobbs “Effective Plastic Shear Area for Tubular Members” Keppel LeTourneau for IOGP TC67/SC7/WG7 Panel 10 20 May 2016

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[3] ISO 19905-1 “Petroleum and natural gas industries — Site-specific assessment of mobile offshore units – Part 1: Jack-ups” 2nd edition, 15th January 2016

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