AN INVESTIGATION INTO THE GLOBAL SWAY BUCKLING FAILURE MODE OF JACK-UP UNITS AND ITS IMPACT ON THE CALCULATION OF EFFECTIVE LENGTH FACTORS FOR INDIVIDUAL LEGS

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ABSTRACT
For jack-up units with single member (as opposed to lattice) legs, the calculation of leg axial strength requires the use of appropriate effective leg buckling length factors (K). A common practice of applying K=2.0 to the 'leg length below keel' has been observed, which appears to stem from a simplification of the leg boundary conditions (BC) at the leg-hull interface ('clamped-roller') and at the footings ('pinned'). This paper follows an investigation into deriving effective length factors using refined case-specific BC's. Focusing on the leg-hull interface we consider how accounting for the stiffness of the global system and the load distribution between legs can lead to more appropriate values of 'K'. The problem is first addressed by undertaking a series of linear eigenvalue buckling analyses of a typical 4 leg jack-up unit, of the type prevalent in the burgeoning Offshore Windfarm industry, using a 'bar stool' finite element model. Firstly, it is demonstrated that surge (or sway) buckling modes are triggered at approximately the same total vertical load level regardless of how that load is distributed between the legs. As the vertical load of an individual leg increases its apparent buckling strength increases, because it receives greater lateral support from the other legs that necessarily become unloaded (noting total vertical load stays approximately constant). This effect is known as 'story buckling' in building frames. Secondly, it is demonstrated that the governing buckling mode is not necessarily surge (or sway) but can be a coupled surge-yaw (or sway-yaw) mode, highlighting the importance of considering the buckling of the structure in 3 dimensions. The results from the eigenvalue analysis are compared against various analytical methods for calculating effective length factors of which the most complex, the AISC Story Stiffness method, is shown to provide a good match with the eigenvalue analysis.

KEY WORDS: Jack-up; sway-yaw coupling; story buckling

INTRODUCTION

The number of single member (tubular and box) leg jack-up units in the world's jack-up fleet has increased substantially over the past 10 years, due mainly to the introduction of Wind Turbine Installation Vessels (WTIV's) to meet the needs of the rapidly expanding offshore wind industry. Whereas in the past single member (as opposed to lattice) leg units were generally confined to coastal construction work, these WTIV's are now operating in deeper water depths and harsher environments as the offshore wind industry begins to exhaust the supply of shallow, benign, sites much as the jack-ups of the oil and gas industry did many years ago. The move towards locating these single member leg units safely in deeper water depths and harsher environments requires the provision of accurate and not unduly conservative site assessment analyses.

The assessment of leg strength is a critical part of the site assessment analysis which for single member legs requires the calculation of an appropriate effective length factor. During the development of JUSTAS (DNVGL's proprietary software for the site assessment of jack-up units) to allow it to perform structural strength analyses for single member leg units, it was found that the advice given by both SNAME [3] and ISO [2] was limited with regards to the calculation of appropriate effective length factors for single member leg units (due to their emphasis on lattice legs). This study was initiated in an attempt to justify the use of less conservative effective length factors (i.e. $K<2$) through rational analysis.

This paper will start by presenting the results of a finite element eigenvalue buckling analysis of a 4-leg unit as a base case. It then goes onto compare the various methods for the calculation of effective length factors with the results of the eigenvalue buckling analysis, starting with the simplest assumptions and building up to the analytical simulation of the eigenvalue analyses. A 4-leg example (rather than a 3-leg example) has been

chosen as the majority of single member leg units have 4 or more legs compared with traditional oil and gas jack-ups which typically have 3 legs.

FINITE ELEMENT LINEAR EIGENVALUE BUCKLING ANALYSIS OF A 4-LEG UNIT

The following representative analysis was carried out to establish a base case which could be used to compare the various methods for the calculation of effective length factors. The details of the model used can be found in the Appendix.

For simplicity the concept of a "moving load" is used schematically to depict the distribution of axial load between the legs, see Figure 1. This is equivalent (for the purposes of buckling analyses) to applying both a point load at the leg centroid and a horizontal load to the unit as shown in Figure 2.

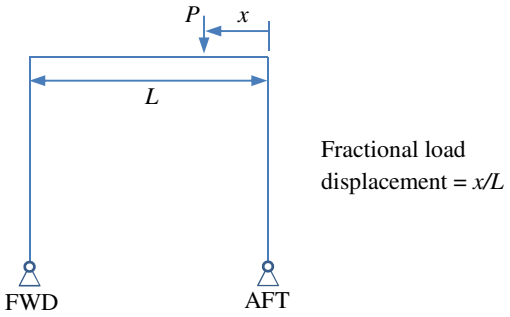


Figure 1 "Moving load", applied in a quasi-static manor

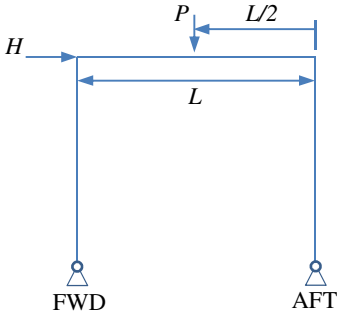
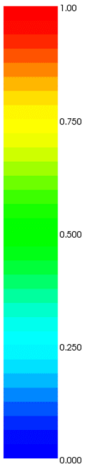
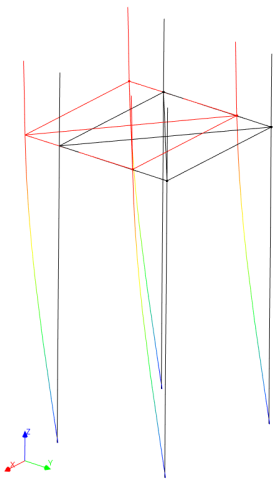
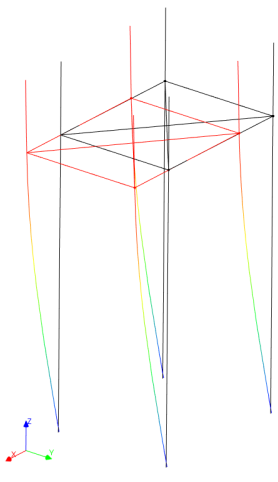
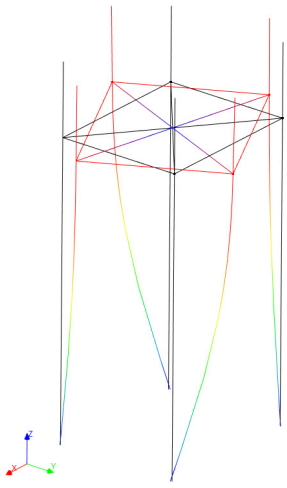


Figure 2 Equivalent combined vertical and horizontal load

Although Figure 1 and Figure *2* above are shown in 2D the structure in the model is 3D with 2 fwd legs and 2 aft legs connected using 4 beams which represent the hull structure.

The following mode shapes were generated from the eigenvalue buckling analysis of the linear elastic weightless structure with the moving load (vertical point load) located at the leg centroid. No yielding effects were accounted for.

Displacement 			
Mode Shape	Mode 1 Pure Sway (y)	Mode 2 Pure Surge (x)	Mode 3 Pure Yaw (R_z)
Buckling load (tonnes)	32,882	33,297	123,392

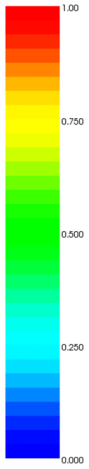
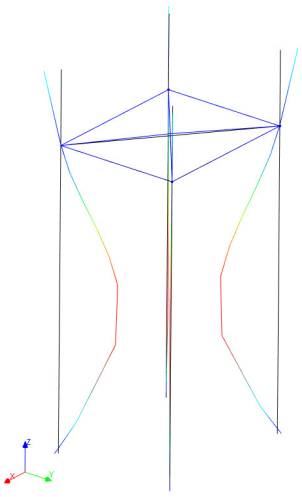
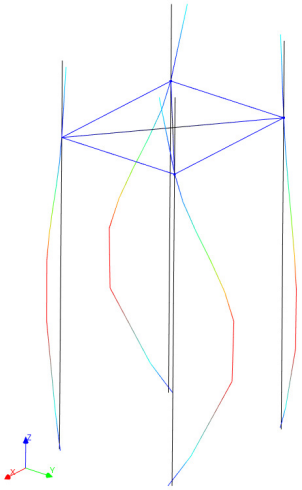
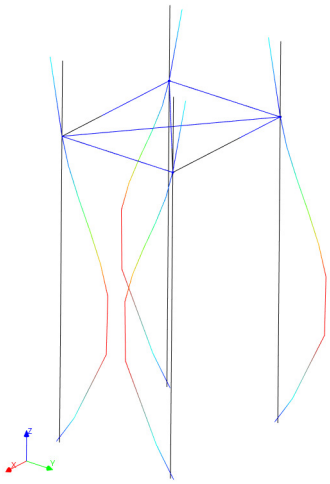
Displacement 			
Mode Shape	Mode 4 Independent leg buckling	Mode 5 Independent leg buckling	Mode 6 Independent leg buckling
Buckling load (tonnes)	278,188	285,820	285,885

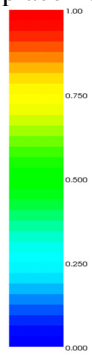
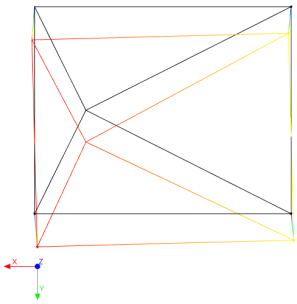
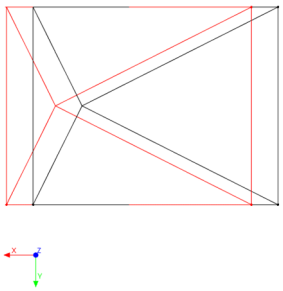
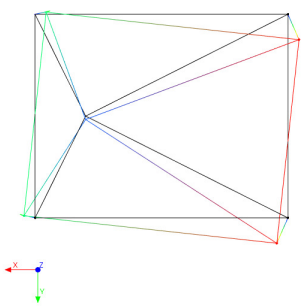
Figure 3 Finite element eigenvalue buckling analysis mode shapes with fractional load displacement of 0.5 (point load at leg centroid)

Observations:

- The transverse leg spacing is marginally smaller than the longitudinal leg spacing which is why the buckling load for the sway mode is marginally less than the buckling load for the surge mode.
- The mode shapes are distinct and there is no interaction between surge, sway and yaw degrees of freedom when both the stiffness of the unit and the loading pattern is symmetric about the *x* and *y* axes.
- Modes 1 and 2 are pure surge and sway modes which are 2D in character. Mode 3 is a pure yaw mode which is 3D in character.

For comparison, the mode shapes shown in Figure 4 were generated from the eigenvalue buckling analysis with a fractional load displacement of 0.8 (see Figure 1) (i.e. point load located towards the fwd legs).

Note, the structure in the following figure is now shown in plan view such that the coupling of sway and yaw degrees of freedom of the hull can be viewed more easily.

Displacement 			
Mode Shape	Mode 1 Coupled Sway (*y*) & Yaw (R_z)	Mode 2 Pure Surge (*x*)	Mode 3 Coupled Yaw (R_z) & Sway (*y*)
Buckling load (tonnes)	30,723 (-6.6%)	33,075 (-0.6%)	151,173 (+22.5%)

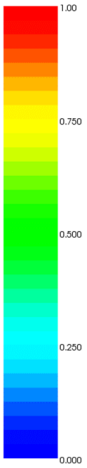
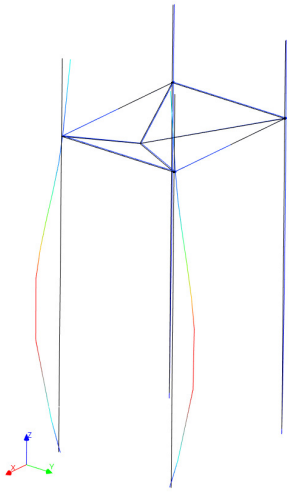
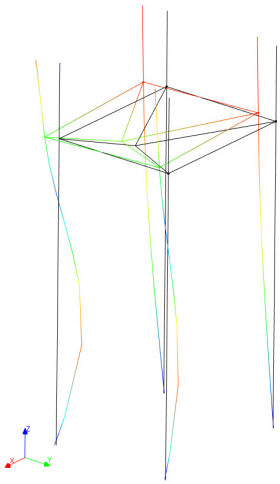
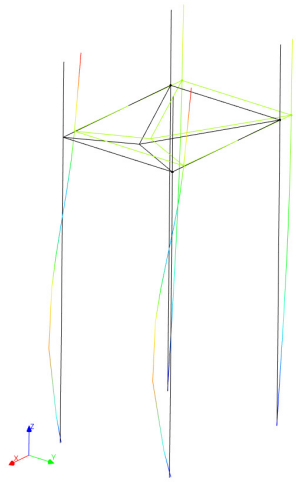
Displacement 			
Mode Shape	Mode 4	Mode 5	Mode 6
Buckling load (tonnes)	177,535 (-36.2%)	186,856 (-34.6%)	187,733 (-34.3%)

Figure 4 Finite element eigenvalue buckling analysis mode shapes with fractional load displacement of 0.8

The percentage given alongside the buckling load represents the change in buckling load between the case with the point load at the leg centroid and the case with the point load at a fractional displacement of 0.8.

From Figure 4 we can see that modes 1 and 3 are now coupled sway-yaw modes with mode 2 remaining a pure surge mode. This coupling effect is due to the geometric stiffness of the fwd legs reducing and the geometric stiffness of the aft legs increasing as the axial load increases in the fwd legs and reduces in the aft legs. This change in stiffness makes the stiffness distribution unsymmetrical about the *y* axis which for mode 1 introduces yaw and for mode 3 introduces sway. The stiffness distribution about the *x* axis remains symmetrical and therefore mode 2 remains a purely surge mode. In this case with a fractional load displacement of 0.8 there is a 6.6% reduction in the total buckling load compared with the case with the point load at the leg centroid. This coupling of modes is not dissimilar to the phenomena of Lateral Torsional Buckling (LTB) of beams.

A comparison of the independent leg buckling modes (4-6) between the two loadcases shows that they are also significantly affected by the position of the point load which in this case results in a reduction in total buckling load of 34-36%. This is mainly due to the significant increase in axial load on the fwd legs along with the coupling of other modes for modes 5 and 6.

If we consider the problem in 2D, the reduction in total buckling load due to the fractional displacement of the point load would be negligible (0.6% in this case) compared with the actual reduction of 6.6% which only becomes apparent when the problem is considered in 3D.

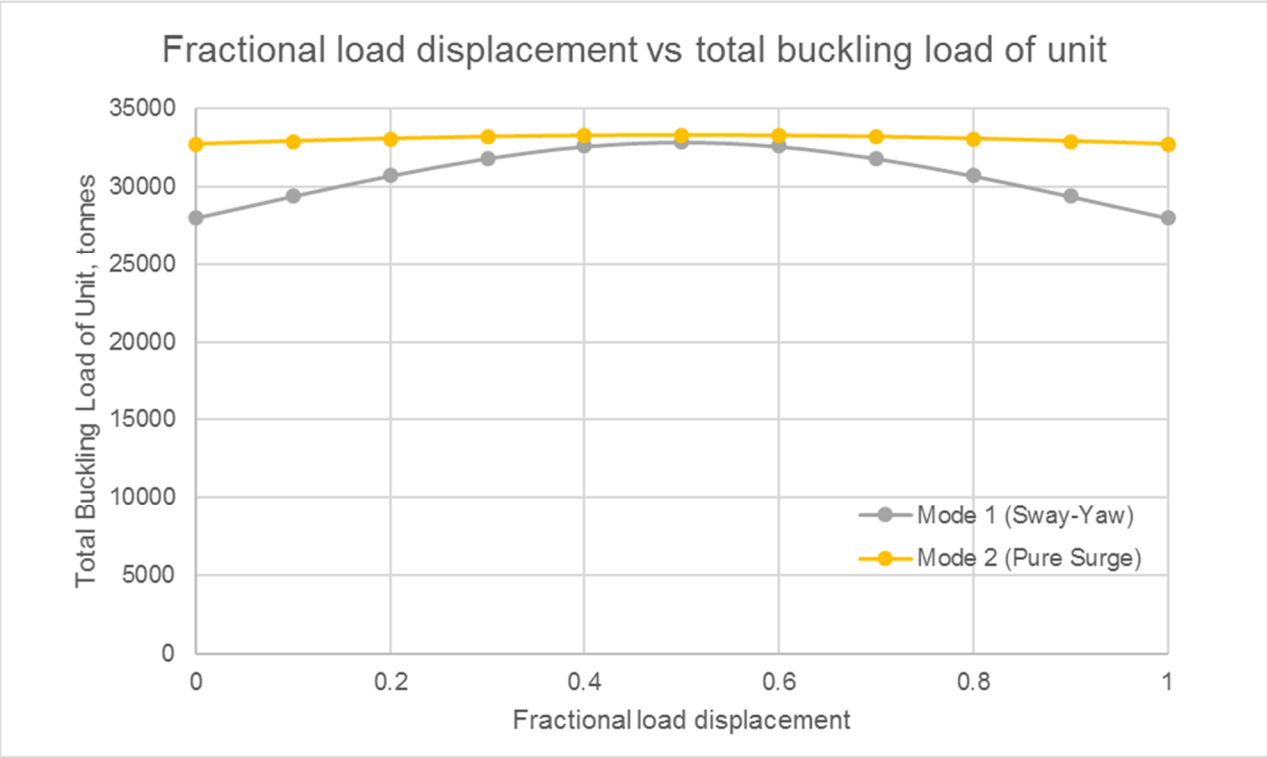


Figure 5 Fractional load displacement vs total buckling load of unit

Figure 5 shows the effect which the position of the moving load has on the total buckling capacity of the unit. The total buckling load for mode 2 remains almost constant regardless of the position of the moving load due to the mode of buckling remaining constant. The total buckling load for mode 1 however varies by about 15% due to the changing contribution of the yaw mode on the predominantly sway mode.

For mode 2 the structure will buckle at approximately the same load regardless of the location of the load and hence regardless of the distribution of vertical loads across the legs. In the extreme case here when the load is at 1.0, all of the vertical load required to buckle the structure is taken by the fwd legs, however the legs do not buckle any sooner than they would have with 50% of the load (load at leg centroid), therefore the apparent sway buckling strength of the fwd legs must increase as the vertical load on the legs increases and conversely the apparent sway buckling strength of the aft legs must decrease as the vertical load on the aft legs decreases. This effect is known in building frames as story buckling behaviour.

To establish the impact of the results illustrated in Figure 5 on the effective length factor, we need to calculate the effective length factors for each of the legs for each of these modes of buckling.

The Euler buckling capacity of any of the legs can be described by:

$$F_{Euler} = \frac{\pi^2 EI}{(KL)^2} \tag{1}$$

We can calculate the Euler buckling capacity of one leg using equation (1) assuming that K=1

$$F_{Euler} = \frac{\pi^2 EI}{L^2} \tag{2}$$

Knowing the load on each leg when the system buckles ($F_{Eigenvalue}$), we can calculate what K should be for each leg to account for all of the effects included in the eigenvalue buckling analysis:

$$F_{Eigenvalue} = \frac{\pi^2 EI}{(KL)^2} = \frac{1}{K^2}\frac{\pi^2 EI}{L^2} = \frac{1}{K^2} F_{Euler} \tag{3}$$

$$K_{Eigenvalue} = \sqrt{\frac{F_{Euler}}{F_{Eigenvalue}}} \quad (4)$$

The effective length factors calculated for the example shown above are presented in Figure 6:

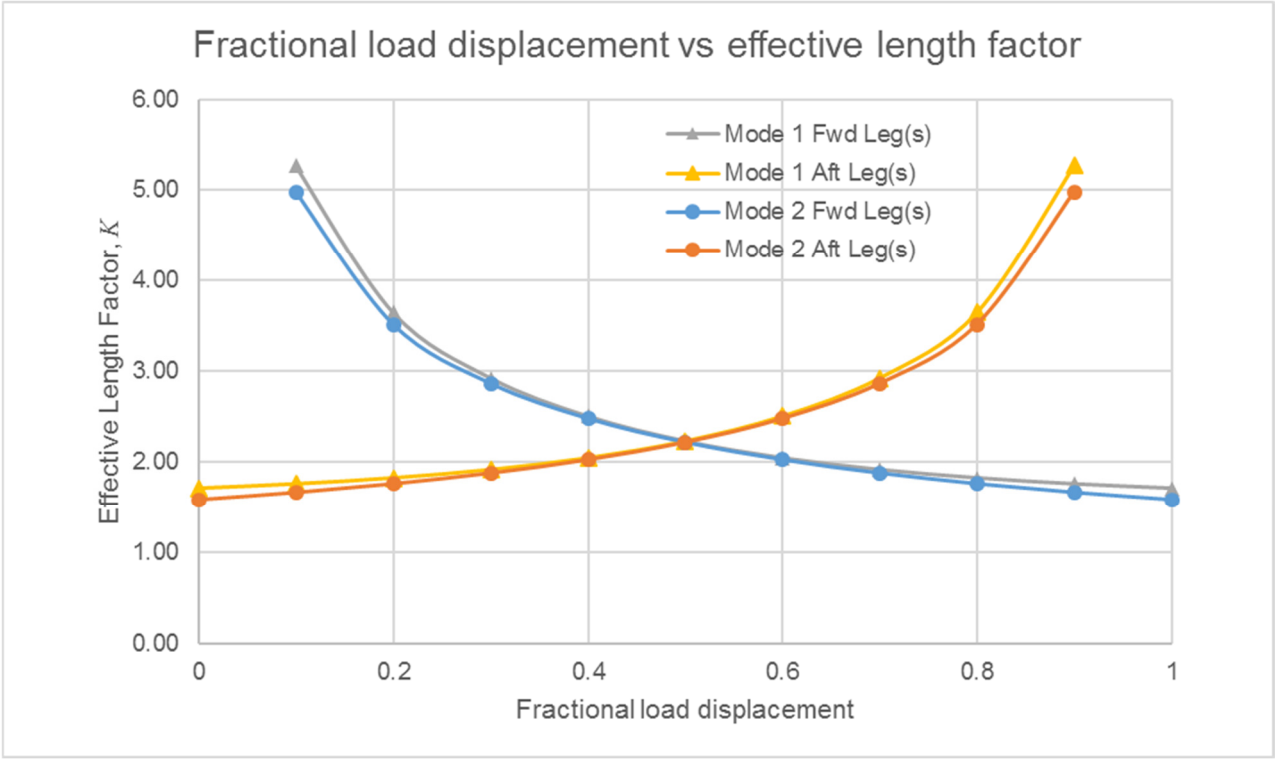


Figure 6 Effective length factors calculated from eigenvalue buckling analysis

It is useful at this point to consider the practical limits which could be imposed on the fractional load displacement. Whilst we are unlikely to see Centre of Gravity shifts of the magnitudes considered in the analyses, horizontal loads due to storm loading can produce much more significant variations in axial load between the legs that are equivalent to the far left and right extremities of the figure (see Figure 2). It is therefore not unrealistic to consider the results for the whole range of fractional load displacement.

The divergence of the results between modes 1 and 2 at the extremities of the figure is due to the effects of coupled sway yaw buckling although these results are less striking than those observed in Figure 5 due to the square root used in the calculation of K.

With the point load at the leg centroid (fractional load displacement = 0.5), K=2.21, this is because we have a pinned foundation acting in conjunction with leg to hull connection stiffness. If the leg to hull connection was rigid, further eigenvalue analysis confirms K=2 as would be expected.

Within the fractional displacement range 0.1 to 0.9, the value of K in Figure 6 ranges from 1.6 (heavily loaded leg(s)) to 5.0 (lightly loaded leg(s)). Beyond the limits of 0.1 and 0.9 the value of K increases towards infinity for the lightly loaded legs.

The eigenvalue analysis was repeated with the "moving load" moving along the Y axis rather than the X axis of the representative unit. Due to the units marginally lower buckling strength about the X axis, the critical mode shape is pure sway for the load at leg centroid and then switches to surge-yaw as the moving load moves away from the centroid, see Figure 7 for an example of the surge-yaw mode shape.

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	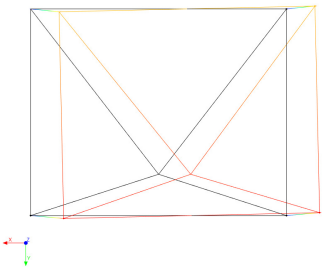	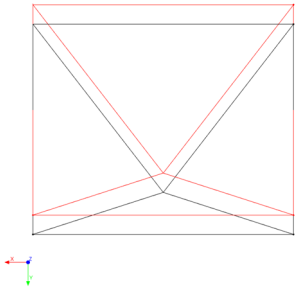	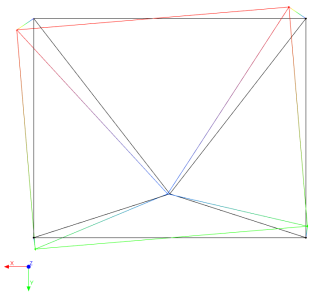
Mode Shape	Mode 1 Coupled Surge (x) & Yaw (R_z)	Mode 2 Pure Sway (y)	Mode 3 Coupled Yaw (R_z) & Surge (x)
Buckling load (tonnes)	31,711	32,703	137,927

Figure 7 Finite element eigenvalue buckling analysis mode shapes with fractional load displacement of 0.8 along Y axis

The effective length factors presented in Figure 6 will form the basis of comparison for the various methods for the analytical calculation of effective length factors presented in the following sections.

SELECTION OF APPROPRIATE *K* FACTORS FOR MEMBER CODE CHECKING

The preceding analysis assesses buckling of the system as a whole, we now progress with the code checking of structural members (legs).

K=2.0

When carrying out code checks on a member basis, both SNAME [3] (section 8.1.4.3) and ISO [2] (section A.13.5-4) recommend using *K*=2.0 unless alternative values are shown applicable by rational analysis.

For the example considered in Figure 6 the use of *K*=2 would be typically conservative for the leeward (loaded) legs and unconservative for the windward (unloaded) legs. Given that the strength of the leeward leg would usually govern, this is likely to result in a conservative strength assessment.

Although our example produces an effective length factor of 2.21 with the point load at the leg centroid; in a practical case this would likely drop to around 2.0 because there is always some level of fixity at the footing which has not been considered in the example.

ALIGNMENT CHARTS

An alternative and commonly used method for the calculation of effective length factors is through the use of the alignment chart presented in ISO [2] Figure A.12.4-1 (originally developed by Kavanagh [4]). This allows the user to take account of the end conditions and structural stiffness of the column in question when determining the effective length factors.

The alignment charts are based on assumptions of idealized conditions such as rigid joints which rarely exist. To account for leg to hull flexibility in jack-up structures it is therefore necessary to adjust the input parameters to account for the relevant assumptions.

G in the following section refers to the AISC stiffness ratio parameter (a ratio of the stiffness of the column in question to the stiffness of the other structural elements which frame into the joints at each end of the column). The subscripts A and B refer to the joints at either end of the column (A = footing, B = leg to hull connection).

In our example the footing is modelled as a friction free pin and hence $G_A = \infty$ (to denote infinite flexibility). G_B is calculated as the ratio of the hull stiffness (modified to account for the leg to hull connection stiffness) and the leg stiffness which in this case results in $G_B = 0.57$ (note the effects of hull sag were not incorporated in the analysis).

Using $G_A = \infty$ and $G_B = 0.57$ an effective length factor of $K \approx 2.2$ can be determined from the alignment chart. This matches very well with the 2.21 calculated from the eigenvalue buckling analysis with the load at the leg centroid.

AISC STORY BUCKLING EQUATIONS

AISC [1] refers to leaning and non-leaning columns (Commentary, Appendix 7, Section 7.2). For application to jack-ups all structural members would be considered in AISC terminology to be non-leaning (in our view).

AISC [1] provides further guidance on the calculation of effective length factors accounting for story stability (although not 3-dimensional coupled mode buckling). The guideline recognises two methods for the calculation of effective length factors accounting for the effects of unsymmetrical story stability: the story stiffness approach (LeMessurier [5] & [6]) and the story buckling approach (Yura [7]). The story stiffness approach is based on the story drift ratio from a first order lateral load analysis at a given applied lateral load level. The story buckling approach is based on the use of the side sway uninhibited alignment chart with appropriate adjustments made to account for non-idealised conditions. Both methods yield effective length factors for each of the columns in the story.

STORY STIFFNESS APPROACH (APPLIED TO JACK-UP UNITS)

The equations presented for the calculation of effective length factors following the story stiffness approach are given below:

$$K_2 = \sqrt{\frac{P_{Story}}{R_M P_r}\left(\frac{\pi^2 EI}{L^2}\right)\left(\frac{\Delta_H}{HL}\right)} \geq \sqrt{\frac{\pi^2 EI}{L^2}\left(\frac{\Delta_H}{1.7 H_{col} L}\right)} \tag{5}$$

$$R_M = 1 - 0.15\left(\frac{P_{mf}}{P_{story}}\right) \tag{6}$$

Taken from AISC [1], pp. 575, equations C-A-7-5 & C-A-7-6 respectively.

K is calculated as the maximum value from the equations on either side of the inequality.

With the associated buckling load calculated as:

$$P_{e2} = \left(\frac{HL}{\Delta_H}\right)\frac{P_r}{P_{story}} R_M \leq \frac{1.7 H_{col} L}{\Delta_H} \tag{7}$$

Taken from AISC [1], pp. 575, equation C-A-7-7

P_{e2} is calculated as the maximum value from the equations on either side of the inequality.

For the purposes of jack-up analyses $R_M = 0.85$ due to the fact that there are no leaning columns.

The equations are made up of two parts, the left-hand side of the inequality describes the situation where all the columns buckle simultaneously in a sidesway mode, the right-hand side of the inequality describes the minimum value that accounts for the interaction between sidesway and non-sidesway buckling (as referred to in AISC, referred to as independent leg buckling earlier in this paper).

To calculate the parameters required to calculate K_2, JUSTAS was used to carry out a first order lateral load analysis of the structure with appropriate vertical and horizontal loads applied to simulate the desired axial

load variation between the legs. The axial load distribution, shear load distribution and drift required to calculate K_2 for each load case was then extracted from the results.

The buckling load calculated using equations (5) and (6) (using the results from the JUSTAS analyses) are plotted against the results of the eigenvalue buckling analysis in Figure 8 and Figure 9.

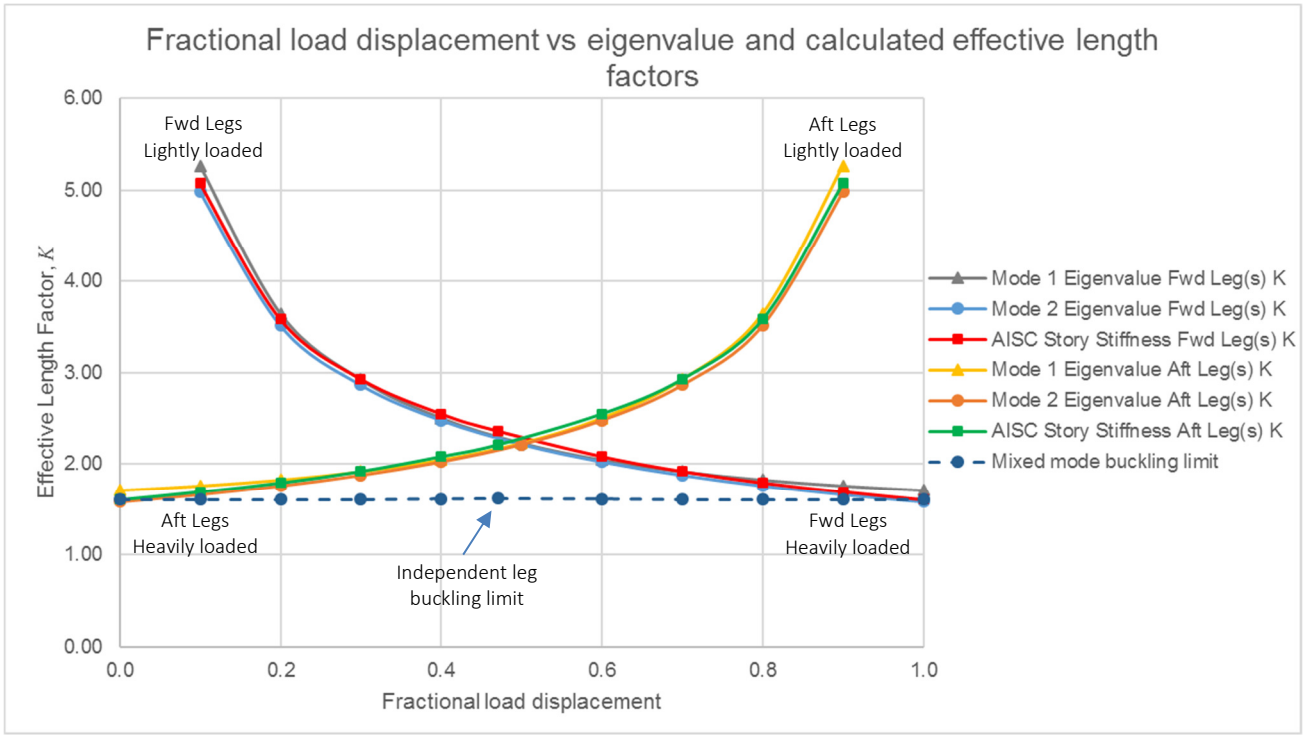

Figure 8 Comparison of effective length factors between eigenvalue buckling analysis and AISC story stiffness approach

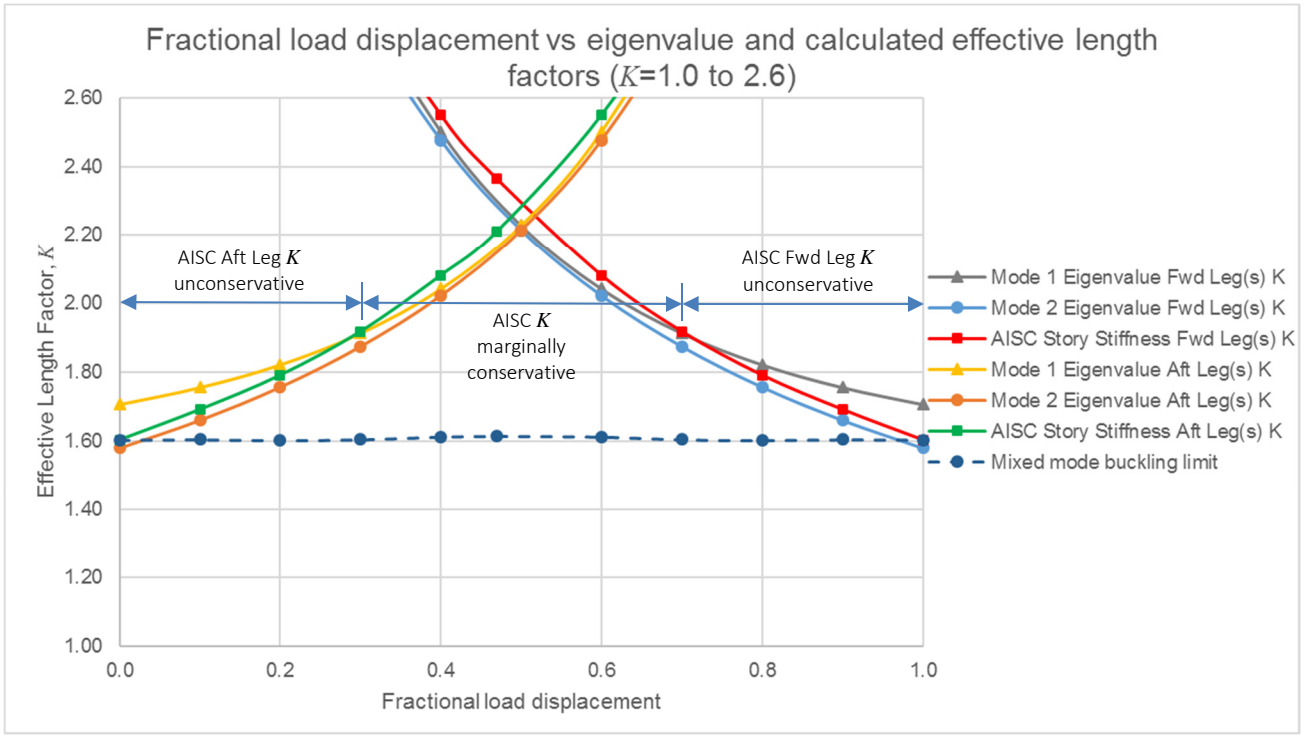

Figure 9 Comparison of effective length factors between eigenvalue buckling analysis and AISC story stiffness approach focusing on K=1.0 to 2.6

Observations:

- The results show an extremely good correlation between the effective length factors calculated from the eigenvalue buckling analysis and those calculated using the AISC story stiffness approach.
- When compared with the results from mode 1 (coupled sway-yaw) of the eigenvalue buckling analysis the calculated effective length factors differ from -6% with the load at 0 or 1 to +2% with the load at 0.5. This indicates that the AISC story stiffness approach is likely to be marginally conservative for lightly horizontally loaded units moving to unconservative for units experiencing larger horizontal loads.
- As would be expected, the calculated effective length factors provide a better match with the mode 2 effective length factors from the eigenvalue analysis which are the result of the 2D pure surge mode rather than the 3D combined sway-yaw mode as is the case with mode 1.

The combined sidesway and non-sidesway bucking limitation (i.e. far right hand term of the K_2 equation) is introduced due to the potential for both sway and non-sway (independent leg) buckling of the columns to occur simultaneously. Referring back to Figure 3 and Figure 4, there is clear separation between sway and independent leg bucking modes with the sway modes (surge/sway and yaw) occurring at significantly lower load levels. It has been found for the representative case in this paper that substantial modification of the stiffness properties is required to promote interaction between sway/surge/yaw and independent leg bucking (Figure 10 shows an example mode shape for this combined mode of buckling). It is expected therefore that this limit will not be encountered for the vast majority of jack-up analyses although further work is required to rule this out.

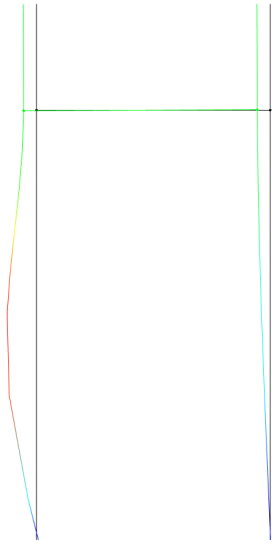

Figure 10 Example of mixed sway and non-sway buckling mode shape

STORY BUCKLING APPROACH

The use of the story buckling approach has been investigated and found to produce similar results to the story stiffness method. Either method could be utilised for the assessment of jack-ups with the story stiffness approach better suited to programmable methods and the story buckling approach better suited to hand calculations (due to its use of the alignment chart).

CONCLUSIONS

This paper presents a comparison of the effective length factors calculated from a series of FE eigenvalue buckling analyses with those calculated using several different methods of varying complexity. The representative case considered is a typical single member leg (as opposed to lattice leg), 4-leg jack-up unit, the details of which are presented in the Appendix.

The comparison has yielded the following observations:

- The use of K=2 for all legs on single member leg jack-ups is broadly conservative for the analysis of leeward legs and unconservative for the analysis of windward legs if the effects of the story buckling phenomena are considered.
- The alignment chart method approach (which still ignores story buckling effects) along with its associated adjustments can be used to produce a more representative effective length factor however it will also be conservative for leeward legs and unconservative for windward legs.
- The eigenvalue buckling analysis indicates that for the representative case, K could be reduced to 1.7 for heavily loaded legs with a pinned foundation (clearly K could be reduced further if the effects of foundation fixity were also considered).
- The application of the AISC Story Stiffness approach (from building frames) to a jack-up unit presented in AISC [1] allows the user to calculate a more accurate effective length factor accounting for the effects of story buckling and therefore providing a much better approximation of the effective length factor for both windward and leeward legs. This method does not however account for the 3D effects of coupled mode buckling (i.e. the coupling of sway-yaw or surge-yaw modes) which leads it to produce unconservative effective length factors (by up to 15% in the representative case considered) for heavily loaded legs.
- An eigenvalue buckling analysis (if available) is the most accurate method for the calculation of effective length factors accounting for the effects of both story buckling and coupled sway-yaw or surge yaw mode shapes. Its use is permitted by ISO [2], SNAME [3] and DNVGL [8] along with a host of other standards.

Further work is recommended to investigate a wider range of units and site parameters than those considered here, including the effects of fixity and the variation of structural stiffness between legs (commonly due to structurally unsymmetrical legs at different orientations). An investigation is also recommended to study the possibility of modifying the AISC story stiffness approach to accommodate the 3D effects of coupled mode buckling.

NOMENCLATURE

E	Young's modulus, taken as 200×10^9 N/m^2;
$F_{Eigenvalue}$	Buckling load of a single leg calculated using an eigenvalue approach;
F_{Euler}	Euler buckling load of a single leg;
H	The sum of the shear loads in all columns (legs);
H_{col}	The shear load in the column (leg) under consideration;
I	Second moment of area of the column (leg) being considered;
K	Effective length factor associated with lateral story buckling;
$K_{Eigenvalue}$	Effective length factor for a single leg calculated using an eigenvalue approach;
L	Length of column being considered (effective leg length);
P_{mf}	The sum of the axial loads in all moment frames (for jack-up analyses this is equal to P_{story} as there are no leaning columns);
P_r	The axial load in the column (leg) under consideration;
P_{story}	The sum of axial loads in all columns (legs);
Δ_H	The sway of the story (hull side sway in the case of jack-ups) calculated from a first order analysis produced by the lateral forces used to compute H.

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APPENDIX – MODEL DETAILS

The following table provides the details of the model used for the representative example in this paper.

Unit Dimensions		
Longitudinal leg spacing	34.44	m
Transverse leg spacing	39.01	m
Effective leg length*	62.63	m
*Leg length between point of effective penetration and leg to hull connection point		
Leg Properties		
Second moment of area (I_{xx})	0.796	m^4
Second moment of area (I_{yy})	0.796	m^4
Hull Properties		
Hull beam second moment of area (I_{xx})	5.400	m^4
Hull beam second moment of area (I_{yy})	5.400	m^4
Leg to hull rotational stiffness (K_{xx})	2.830	t-m/rad
Leg to hull rotational stiffness (K_{yy})	2.830	t-m/rad
Leg to footing rotational stiffness (K_{xx})	Pinned	
Leg to footing rotational stiffness (K_{yy})	Pinned	
General Properties		
Young's modulus	$200x10^9$	N/m^2