ON JACK-UP HULL-IN-WATER WAVE-INDUCED LEG LOADS AND SURGE MOTION DURING EXTRACTION OF EMBEDDED SPUDCANS

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ABSTRACT
A simplified frequency domain model is developed for the analysis of jack-up hull-in-water wave-induced leg loads and surge motion so that limiting seastates can be defined for the safe extraction of embedded spudcans. Prediction of wave-induced leg loads can be used to safeguard the legs and associated holding systems whilst knowledge of surge motion is especially important when the removal operation is taking place next to an adjacent fixed platform. Consideration is given to hydrodynamic loading effects on a rigid hull due to wave diffraction and radiation coupled with elasticity effects due to leg and holding system flexibility. The cases of one and three legs with embedded spudcans are considered for a representative rig in 70m and 90m water depths. Recognition is given to the Haskind relations between wave excitation forces and radiation damping for large volume bodies such as the hull of a jack-up. Significant radiation damping due to hull motion is observed for all the modes.

KEY WORDS: jack-up; leg extraction; embedded spudcan; wave diffraction; wave radiation; Haskind relations.

INTRODUCTION

The purpose of this paper is to propose a simplified frequency domain model that is relevant to the analysis of jack-up hull-in-water wave-induced leg loads and surge motion in order that limiting seastates can be defined for safe extraction of embedded spudcans. Prediction of wave-induced leg loads can be used to safeguard the legs and associated holding systems whilst knowledge of surge motion is especially important when the removal operation is taking place next to an adjacent fixed platform.

Progress has recently been made in the determination of the limiting conditions for jack-up units coming off location. Dowdy et al. [1] presented a 3-stage approach which included measuring wave data on site, assessing the real-time hull response and checking against the pre-determined limits in order that GO/NO-GO decisions can be made for coming off location operations. The analytical models adopted were calibrated against data from actual rig moves. Carre et al. [2] discussed various analyses and techniques which have been developed to optimize rig move performance during going on location, preloading and coming off location. Chakrabarti et al. [3] presented an analysis procedure of jack-up during leg extraction, which considered the wave diffraction effects, buoyancy springs and various bottom conditions to represent the process of leg extraction, i.e. three legs pinned, one leg free with the other two legs stuck and two legs free with the other one leg stuck. However, the radiation damping was considered to be negligible.

In this paper, consideration is given to hydrodynamic loading effects on a rigid hull due to wave diffraction and radiation coupled with elasticity effects due to leg and holding system flexibility. The cases of one and three legs with embedded spudcans are considered for a representative rig in 70m and 90m water depths. Recognition is given to the Haskind relations between wave excitation forces and radiation damping for large volume bodies such as the hull of a jack-up. The significance of the radiation damping due to hull motion is assessed.

THEORETICAL FORMULATION

The response of a large floating body undergoing small amplitude motions in low to moderate sea conditions can be analysed using linear spectral methods. An inviscid, incompressible fluid undergoing irrotational flow is typically assumed for the hydrodynamic analysis. Linear transfer functions (i.e. response amplitude operators) describing the responses in regular waves provide the main building block for the analysis. The equation describing the steady state response of a rigid floating body in a regular wave has the following form:

$$\{-\omega^2[\boldsymbol{M} + \boldsymbol{A}(\omega)] - i\omega \boldsymbol{B}(\omega) + \boldsymbol{K}\}\ \boldsymbol{X}(\omega) = \boldsymbol{F}(\omega) \quad (1)$$

where, $\boldsymbol{M}$ is the mass matrix; $\boldsymbol{A}(\omega)$ is the frequency-dependent added mass matrix; $\boldsymbol{B}(\omega)$ is the frequency-dependent radiation damping matrix; $\boldsymbol{K}$ is the stiffness matrix representing restoring effects; $\boldsymbol{X}(\omega)$ is the frequency-dependent vector of complex response amplitudes for the rigid body degrees of freedom; $\boldsymbol{F}(\omega)$ is the frequency-dependent complex amplitude vector of wave excitation loads due to the combined incident and diffracted wave field; and here $i = \sqrt{-1}$.

SIMPLIFIED STRUCTURAL MODEL

A jack-up rig with the hull in the water with one leg bottom-pinned can be considered as a simplified two-dimensional linear elastic system as illustrated in Figure 1. The model closely resembles that used in [4, 5] for the investigation of spudcan impacts when going on location.

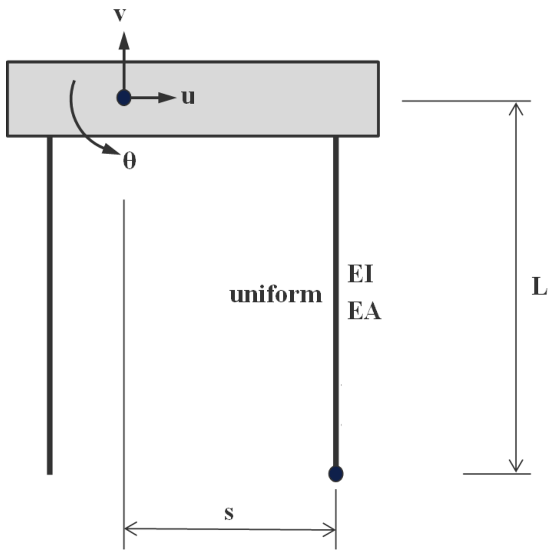


Figure 1 Schematic of simplified 2-D linear elastic structural model

A rigid hull is connected to a single leg which has cross-sectional area A and second moment of area I. The leg material has a modulus of elasticity E. The horizontal offset of the leg from the hull centre of gravity is s and the length of the leg is L. The leg is rigidly attached to the hull and is pinned at the seabed. The model has three degrees-of-freedom: surge, heave and pitch of the hull (u, v, θ). The mass matrix is of the following form:

$$\boldsymbol{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_{total} \end{bmatrix} \quad (2)$$

where m and I_{total} are the total mass and moment of inertia for the hull and legs.

The hull has hydrostatic restoring stiffness k_v in heave and k_θ in pitch. The stiffness k_h represents the horizontal restoring effects induced by leg pulling. It may be shown that this hull-in-water model, with single leg bottom-pinned restraint, has the following stiffness matrix:

$$\boldsymbol{K} = \begin{bmatrix} \left(\frac{3EI}{L^3} + k_h\right) & 0 & \frac{3EI}{L^2} \\ 0 & \left(\frac{EA}{L} + k_v\right) & \frac{EAs}{L} \\ \frac{3EI}{L^2} & \frac{EAs}{L} & \left(\frac{3EI}{L} + \frac{EAs^2}{L} + k_\theta\right) \end{bmatrix} \tag{3}$$

The model can be readily extended to the case of three legs pinned by incorporating two legs at a distance $s/2$ aft of the hull centre of gravity:

$$\boldsymbol{K} = \begin{bmatrix} \left(\frac{9EI}{L^3} + k_h\right) & 0 & \frac{9EI}{L^2} \\ 0 & \left(\frac{3EA}{L} + k_v\right) & 0 \\ \frac{9EI}{L^2} & 0 & \left(\frac{9EI}{L} + \frac{3EAs^2}{2L} + k_\theta\right) \end{bmatrix} \tag{4}$$

STRUCTURAL RESPONSE AND LEG LOAD IN RANDOM WAVES

In the jth degree of freedom, the first order wave force at the frequency of the incident waves may be written in the form:

$$F_j(\omega) = T_j(\omega)\zeta_0(\omega) \tag{5}$$

where $\zeta_0(\omega)$ is the complex wave amplitude of the incident wave with frequency ω and $T_j(\omega)$ here are the corresponding force transfer functions.

As the system is assumed to be linear, the first order response can be easily obtained from the equation of motion, written as:

$$X_j(\omega) = R_j(\omega)\zeta_0(\omega) \tag{6}$$

where:

$$\begin{bmatrix} R_1(\omega) \\ R_2(\omega) \\ R_3(\omega) \end{bmatrix} = \{-\omega^2[\boldsymbol{M} + \boldsymbol{A}(\omega)] - i\omega\boldsymbol{B}(\omega) + \boldsymbol{K}\}^{-1} \begin{bmatrix} T_1(\omega) \\ T_2(\omega) \\ T_3(\omega) \end{bmatrix} \tag{7}$$

$R_j(\omega)$ is the linear transfer function of the hull motion in surge, heave and pitch when $j = 1,2,3$ respectively, representing the relationship between the incident wave amplitude and its corresponding response.

The bow leg axial force and bending moment at the lower guide level may be written as:

$$\mathrm{F}_{\mathrm{axial}}(\omega) = \mathrm{T}_{\mathrm{axial}}(\omega)\zeta_0(\omega) \tag{8}$$

$$\mathrm{F}_{\mathrm{moment}}(\omega) = \mathrm{T}_{\mathrm{moment}}(\omega)\zeta_0(\omega) \tag{9}$$

where $\mathrm{T}_{\mathrm{axial}}(\omega)$ and $\mathrm{T}_{\mathrm{moment}}(\omega)$ are the linear transfer functions of the bow leg axial force and bending moment at the lower guide level, respectively. They can be given in the form:

$$\mathrm{T}_{\mathrm{axial}}(\omega) = \frac{EA[R_2(\omega) + sR_3(\omega)]}{L} \tag{10}$$

$$\mathrm{T}_{\text{moment}}(\omega) = \frac{3EI[R_1(\omega) + LR_3(\omega)]}{L^2} \tag{11}$$

The system is driven by the wave force arising from the random waves which may be presented by a single-sided wave spectrum $\mathrm{S}(\omega)$. Following the conventional spectral analysis [6], the first order response spectra in the jth degree of freedom can be given by:

$$\mathrm{S}_{X_j}(\omega) = \left|R_j(\omega)\right|^2 \mathrm{S}(\omega) \tag{12}$$

The variance of the response can be then computed in a standard way:

$$\sigma_{X_j}{}^2 = \int_0^\infty \mathrm{S}_{xx_j}(\omega) d\omega \tag{13}$$

where σ_{X_j} is the standard deviation of the first order response in the jth degree of freedom.

The most probable maximum extreme (MPME) responses are based upon 1000 peaks of a linear narrow band Gaussian random process, thereby giving the usual factor of 3.72 to be applied to the standard deviations of responses (i.e. $\sqrt{2\ln 1000}$). Thus the MPME of the hull response in the jth degree of freedom may be written as:

$$\mathrm{MPME}_{xx_j} = 3.72\sqrt{\int_0^\infty \left|R_j(\omega)\right|^2 \mathrm{S}(\omega) d\omega} \tag{14}$$

Similarly, the MPME of the bow leg axial force and bending moment at the lower guide level can be obtained from:

$$\mathrm{MPME}_{\mathrm{F}_{\text{axial}}} = 3.72\sqrt{\int_0^\infty |\mathrm{T}_{\text{axial}}(\omega)|^2 \mathrm{S}(\omega) d\omega} \tag{15}$$

$$\mathrm{MPME}_{\mathrm{F}_{\text{moment}}} = 3.72\sqrt{\int_0^\infty |\mathrm{T}_{\text{moment}}(\omega)|^2 \mathrm{S}(\omega) d\omega} \tag{16}$$

HYDRODYNAMIC LOADING FOR A REPRESENTATIVE HULL

Hydrodynamic coefficients including the added mass $\boldsymbol{A}(\omega)$, radiation damping $\boldsymbol{B}(\omega)$ and force transfer function $T_j(\omega)$ as shown in Equation (7), can be readily obtained over a finite range of frequencies using conventional three-dimensional panel codes. Figure 2 shows the panel mesh for a hydrodynamic model of a representative hull which has been previously developed [4, 5]. The main simplification compared to a more realistic model of a jack-up rig is the omission of leg wells. 30 different frequencies over the range 0.105 to 1.795 rad/s (0.0167 to 0.2857 Hz) for wave direction from 0^0 to 180^0 using AQWA [7] are considered in this work.

Only on-diagonal added mass and damping coefficients for surge, heave and pitch are incorporated in the model. For this study, the only source of damping considered is that due to wave radiation damping; fluid viscous damping and soil damping have been neglected.

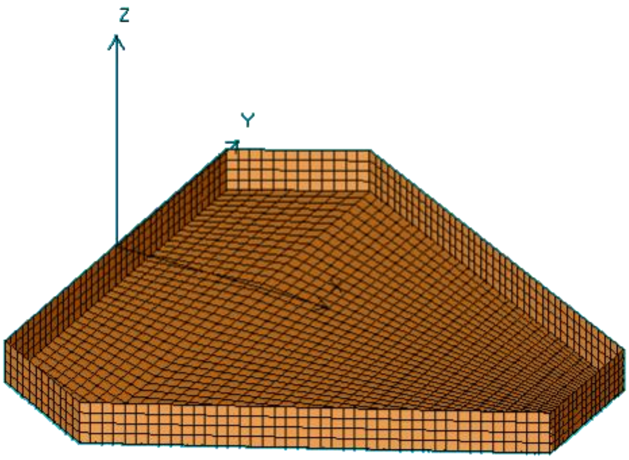


Figure 2 Hydrodynamics panel model (AQWA plot)

NUMERICAL EXAMPLES

Details of the model for a representative jack-up unit are given in Table 1. The cases of one and three legs with embedded spudcans are considered for 70m and 90m water depths.

Table 1 Model parameters

Hull length (m)	70.36
Hull width (m)	76.00
Hull depth (m)	9.40
Hull draught (m)	6.00
Hull waterplane area (m^2)	3,620
Hull second moment of waterplane area (m^4)	1.22×10^6
Hull and leg dry mass (tonnes)	22,265
Longitudinal leg spacing (m)	45.72
Transverse leg spacing (m)	47.55
Leg cross-sectional area (m^2)	0.537
Leg second moment of area (m^4)	15.373
Leg length, L (m)	70.00 & 90.00
Leg offset, s (m)	30.48
Radius of gyration for hull and legs (m)	37.69

The sea-state is described by a Pierson-Moskowitz wave spectrum, which is representative of a fully developed Sea. The formulation is expressed as a function of significant wave height H_s and the average wave period T_1 [8]:

$$S(\omega) = {H_s}^2 T_1 \frac{0.11}{2\pi}\left(\frac{\omega T_1}{2\pi}\right)^{-5} exp\left[-0.44\left(\frac{\omega T_1}{2\pi}\right)^{-4}\right] \tag{17}$$

where T_1 can be written as:

$$T_1 = 1.086 T_2 \tag{18}$$

T_2 is the zero crossing wave period and its relation with the peak wave period T_0 is given by:

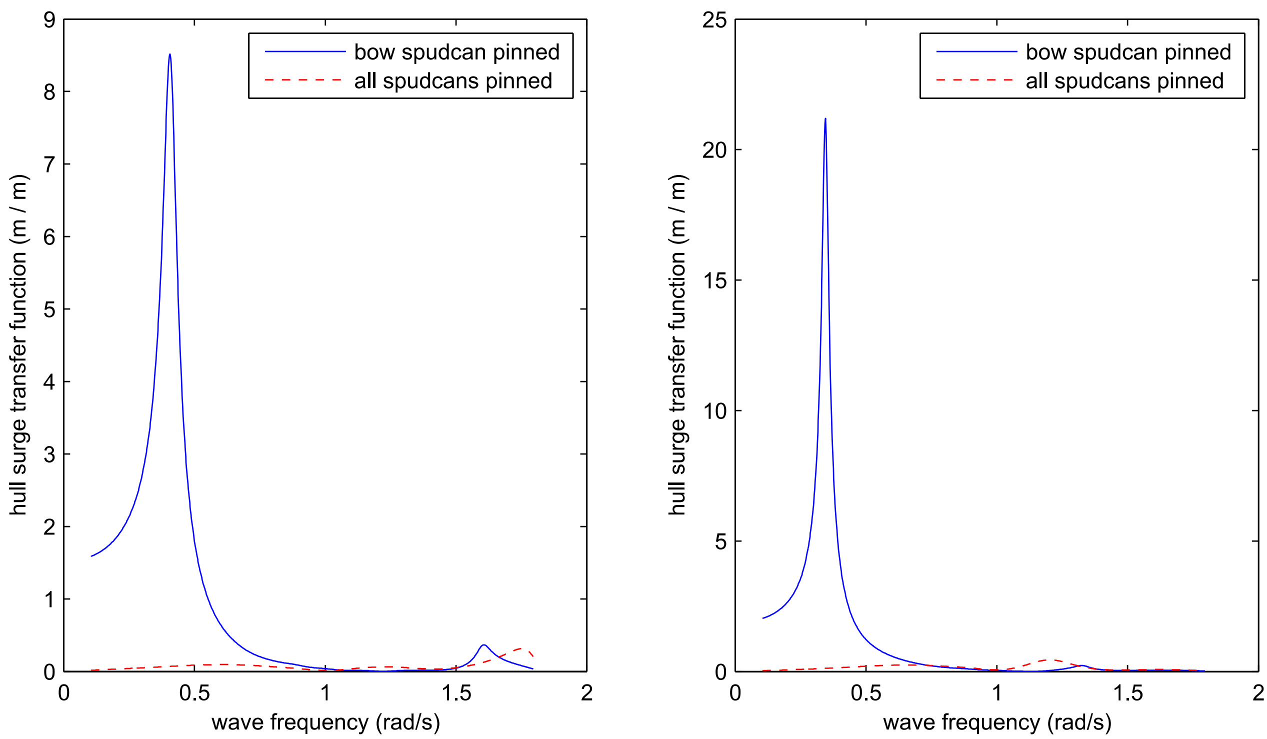
$$T_0 = 1.408 T_2 \tag{19}$$

For illustration, the representative rig in head seas is considered. The stiffness matrices for the cases of one and three legs with embedded spudcans are evaluated by Equations (3) and (4). They are then used to calculate the linear transfer functions of the hull surge motion, the bow leg axial force and bending moment at the lower guide level by Equations (7), (10) and (11). The MPMEs of the hull surge motion and leg loads are computed by Equations (14), (15) and (16).

Figure 3 shows the linear transfer functions for hull surge motion in the cases of 70m and 90m water depths. In the case of 70m water depth when the bow spudcan is pinned there are two resonant peaks within the frequency range shown: one at around 0.41 rad/s (15.3s period) associated with a rocking mode about the spudcan and another at 1.61 rad/s (3.9s period) associated with leg-to-hull bending. When all spudcans are pinned there is one resonant peak that can be seen at around 1.75 rad/s (3.6s period) and this is associated with typical portal frame action. Similarly, resonant peaks at 0.35 rad/s (18.1 period) and 1.32 rad/s (4.8s) are seen for the rocking mode and leg-to-hull bending mode in the case of 90m water depth when the bow spudcan is pinned. When all spudcans are pinned, one resonant peak at 1.20 rad/s (5.2s period) is seen for the portal frame mode.

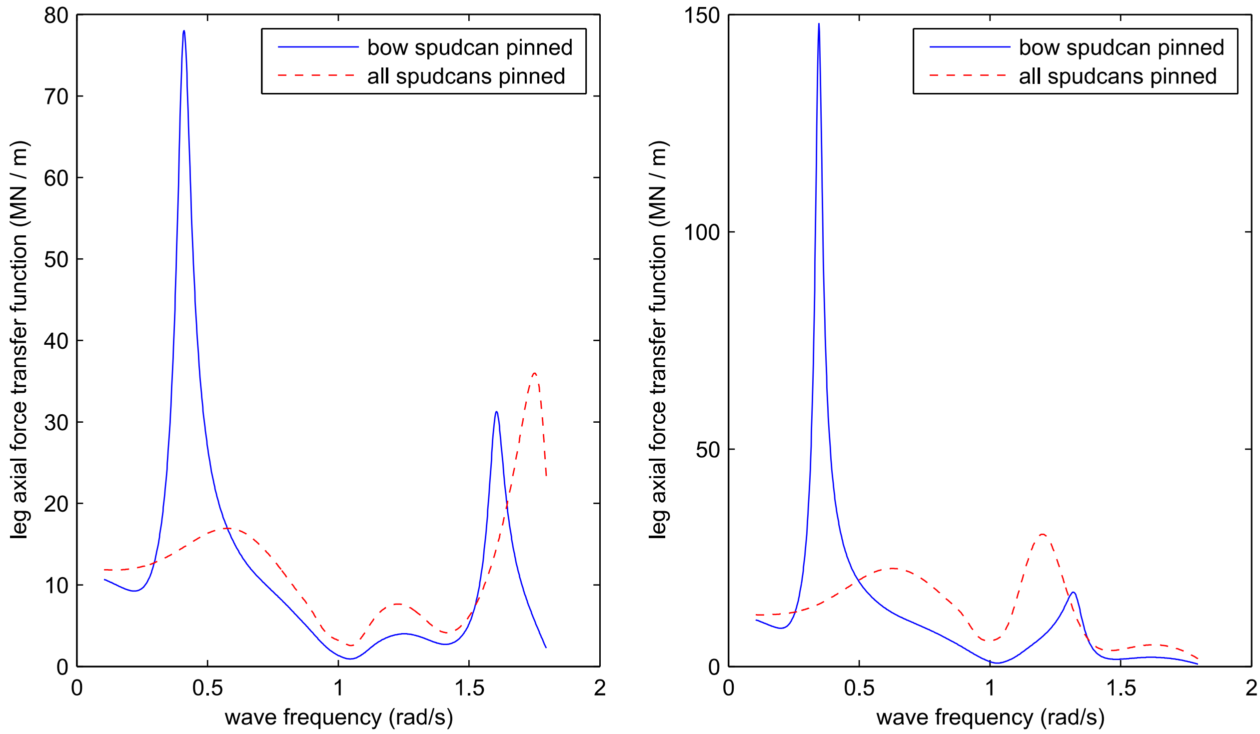
Figures 4 and 5 show the corresponding linear transfer functions of leg axial force and bending moment at the lower guide level in the bow leg. Structural resonant responses in the leg-to-hull bending mode (bow spudcan pinned) and portal frame mode (all spudcans pinned) are clearly seen. Based on the half-power bandwidth method, critical damping ratios of 6%, 2% and 3% are observed for the rocking mode and leg bending mode (bow spudcan pinned) and the portal frame mode (all spudcans pinned) in the 70m water depth case. In the 90m water depth case, critical damping ratios of 4%, 3% and 6% are observed for the rocking mode and leg bending mode (bow spudcan pinned) and the portal frame mode (all spudcans pinned) respectively. It indicates that the levels of wave radiation damping are significant in all the modes. In order to further investigate the significance of radiation damping, the Haskind relations between wave excitation forces and radiation damping for large volume bodies are considered in the Appendix. It is shown therein that the rocking modes (longer periods) are less sensitive to wave direction whilst the leg bending modes are highly sensitive to wave direction.

Table 2 and Table 3 show the most probable maximum extremes (MPME) in various seastates covering a range of wave periods in the cases of 70m and 90m water depths. It can be seen that the hull surge responses and leg bending moments are most pronounced in seastates with longer wave periods and are dominated by the rocking mode that arises when only the bow spudcan is pinned. This implies that the long wave period swell waves attempt to excite the rocking mode, causing significant hull surge response and leg bending moment during leg extraction.



a. 70m water depth   b. 90m water depth

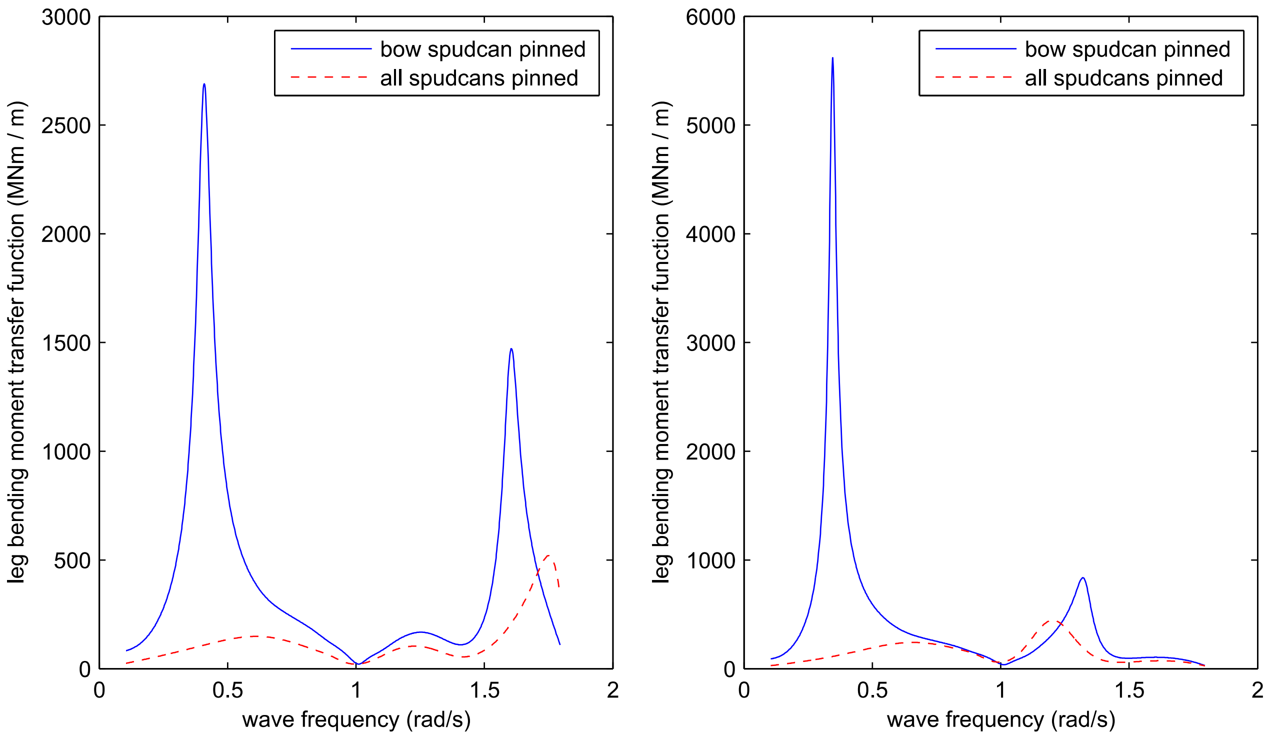
Figure 3 Linear transfer function of hull surge motion



a. 70m water depth

b. 90m water depth

Figure 4 Linear transfer function of leg axial force



a. 70m water depth

b. 90m water depth

Figure 5 Linear transfer function of leg bending moment

Table 2 MPME responses in two extraction scenarios for 70m water depth case

70m water depth			Bow spudcan pinned			All spudcans pined		
H_s	T_2	T_0	MPME hull surge motion	MPME leg axial force	MPME leg bending moment	MPME hull surge motion	MPME leg axial force	MPME leg bending moment
m	s	s	m	MN	MNm	m	MN	MNm
2.0	2.5	3.5	0.164	13.8	647	0.181	20.8	300
2.0	3.0	4.2	0.183	15.7	736	0.190	21.9	314
2.0	4.0	5.6	0.155	12.8	586	0.149	17.7	246
2.0	5.0	7.0	0.243	12.3	470	0.126	17.3	206
2.0	6.0	8.4	0.594	16.5	505	0.132	20.9	212
2.0	7.0	9.9	1.538	25.4	758	0.144	24.3	228
2.0	8.0	11.3	3.346	40.5	1281	0.150	26.4	236
2.0	9.0	12.7	5.342	56.4	1851	0.150	27.4	236
2.0	10.0	14.1	6.836	67.2	2249	0.147	27.6	230
2.0	11.0	15.5	7.668	71.8	2434	0.142	27.4	222
2.0	12.0	16.9	7.969	72.0	2458	0.136	27.0	211
2.0	13.0	18.3	7.926	69.4	2382	0.129	26.5	201
2.0	14.0	19.7	7.689	65.4	2253	0.123	26.1	191
2.0	15.0	21.1	7.355	60.9	2103	0.117	25.6	181

Table 3 MPME responses in two extraction scenarios for 90m water depth case

90m water depth			Bow spudcan pinned			All spudcans pined		
H_s	T_2	T_0	MPME hull surge motion	MPME leg axial force	MPME leg bending moment	MPME hull surge motion	MPME leg axial force	MPME leg bending moment
m	s	s	m	MN	MNm	m	MN	MNm
2.0	2.5	3.5	0.067	4.3	215	0.103	6.9	103
2.0	3.0	4.2	0.118	8.8	432	0.247	16.8	248
2.0	4.0	5.6	0.145	10.9	517	0.372	25.9	372
2.0	5.0	7.0	0.245	11.7	470	0.375	28.2	373
2.0	6.0	8.4	0.513	15.2	497	0.389	31.9	384
2.0	7.0	9.9	1.009	20.4	622	0.401	34.8	393
2.0	8.0	11.3	2.192	28.9	920	0.399	36.0	389
2.0	9.0	12.7	4.897	46.0	1586	0.385	35.8	375
2.0	10.0	14.1	8.529	69.0	2494	0.365	35.0	354
2.0	11.0	15.5	11.784	89.2	3294	0.343	33.7	332
2.0	12.0	16.9	14.002	102.1	3814	0.321	32.4	310
2.0	13.0	18.3	15.163	107.8	4054	0.300	31.2	289
2.0	14.0	19.7	15.513	108.1	4086	0.280	30.0	269
2.0	15.0	21.1	15.325	105.0	3983	0.262	28.9	251

CONCLUSIONS

This work has presented a simplified frequency domain model that might be used to assess wave-induced hull motions and leg loads during extraction of embedded spudcans. As expected, the case of one leg being pinned and the other legs free is the most critical condition, not only for hull motions but also wave-induced leg loads. For the cases considered, structural resonant responses in the rocking mode and leg-to-hull bending mode (bow spudcan pinned) and portal frame mode (all spudcans pinned) have been clearly seen. Significant radiation damping due to hull motion has been observed for all the modes. Recognition has been given to the Haskind relations between wave excitation forces and radiation damping for large volume bodies such as the hull of a jack-up. Hull surge responses and leg bending moments have been shown to be most pronounced in seastates with longer wave periods and to be dominated by the rocking mode that arises when only the bow spudcan is pinned. This implies that the long wave period swell waves attempt to excite the rocking mode, causing significant hull surge response and leg bending moment during leg extraction.

In further work, it is recommended that detailed consideration be given to the following matters:

- leg-to-hull flexibility;
- leg shear flexibility;
- soil damping of loosened legs/spudcans;
- hydrodynamic damping due to viscous effects;
- lateral soil stiffness of loosened spudcans;
- spudcan fixity for embedded spudcans;
- three-dimensional structural model (i.e. 6 x 6 stiffness matrix);
- hydrodynamic modelling of leg wells, including viscous damping effects.

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APPENDIX – APPLICATION OF THE HASKIND RELATIONS TO LIGHTLY DAMPED MODES IN RANDOM WAVES

The radiation damping coefficient B_{jj} associated with the jth degree of freedom (or mode) is related to the wave exciting force through the Haskind relations [9] as follows:

$$B_{jj} = \frac{k}{8\pi\rho g V_g} \int_0^{2\pi} \left|T_j(\theta)\right|^2 d\theta \qquad (A1)$$

where $T_j(\theta)$ here is the amplitude of the wave exciting force for regular waves of direction θ and unit amplitude; k is the wave number, V_g is the group velocity and other terms have their usual meaning.

It may be shown using a white noise approximation [10] that the variance of response for a lightly damped mode in random waves is proportional to the spectral density of the excitation force and inversely proportional to the damping coefficient at the natural frequency of the mode. Thus, we apply the Haskind relations to the following ratio:

$$\frac{\left|T_j(\beta)\right|^2}{B_{jj}} \propto \frac{1}{I_j(\beta)} \qquad (A2)$$

where β is the particular wave direction being considered and

$$I_j(\beta) = \int_0^{2\pi} \left|T_j(\theta)/T_j(\beta)\right|^2 d\theta \qquad (A3)$$

Table A1 shows the values obtained from (A3) for head seas (i.e. $\beta = \pi$) and stern seas (i.e. $\beta = 0$) in the cases of a vertical body of revolution and a typical jack-up hull. Unlike a vertical body of revolution, the values of $I_j(\beta)$ for the jack-up hull are strongly dependent upon wave period. At the longer wave periods of 10s and 15s the values are comparable with those of a vertical body of revolution. However, with shorter wavelengths and increased diffraction effects at the shorter period of 4s for stern seas the values of $I_j(0)$ are significantly reduced. It follows that, for stern seas, the dynamic response of a lightly damped mode of around 4s period is expected to be significantly larger for a typical jack-up than it would be for a similar sized hull formed from a truncated vertical circular cylinder. On the other hand, for head seas, the dynamic response of a lightly damped mode of around 4s period is expected to smaller for a typical jack-up than it would be for a similar sized hull formed from a truncated vertical circular cylinder. Thus, in general we expect the rocking modes (longer periods) to be less sensitive to wave direction whilst the leg bending modes will be highly sensitive to wave direction.

Table A1 Sensitivity of $I_j(\beta)$ to wave period T and hull geometry

Case	Head seas		Stern seas	
	$I_1(\pi)$ surge	$I_2(\pi)$ heave	$I_1(0)$ surge	$I_2(0)$ heave
Vertical body of revolution for $0 < T < \infty$	π	2π	π	2π
Jack-up hull for T = 15s	3.185	6.357	3.143	6.357
Jack-up hull for T = 10s	3.781	6.578	2.808	6.692
Jack-up hull for T = 4s	4.222	32.488	0.968	1.879