TIME-DOMAIN DYNAMIC AMPLIFICATION (TDAF) ANALYSES WITH LINEAR IRREGULAR WAVES

J. E. Ottersen and L. Shang
DNV AS

ABSTRACT
The dynamic amplification of the wave loads is a significant part of the global loads for jackups and there are various methods to estimate the amplification factor (DAF). The DNV Sesam software package supports time-domain wave load analyses with linear irregular waves and a linear dynamic structural solver. Time-domain analyses capture the real dynamic behaviour more accurate than deterministic analyses, including the inherent randomness of a seastate. This work presents how such analyses could be performed, including a comparison with results from other methods to calculate the DAF, as "EDAF" [4] and "SDOF" [2].

The procedure is containing a screening phase for the governing storm condition and thereafter a sufficient number of realisations of 3-hour storm conditions to converge the amplification factor or the inertia load and moment.

KEY WORDS: Jack-up dynamics, inertia, time-domain, Sesam software, DAF

1. INTRODUCTION

The dynamic behaviour of offshore structures has been studied for decades. This is especially important for self-elevated jackup rigs, as they typically have a global natural period of 6-12 seconds, which is close to the typical wave excitation period. When exposed to waves, the total structural response is larger than those caused by the static wave load alone. The additional structural response is caused by the dynamic amplification and can be represented as an additional inertia load/moment or an acceleration field. It is required in DNV Offshore structural rules, ref [1], that the inertia load from dynamic effects shall be included. One commonly used approach is to estimate the dynamics considering the jackup as a Single Degree of Freedom System (SDOF). Despite the simplifications assumed in the SDOF method, it has been accepted by the industry as a simple method which gives reasonable results.

There exist a few alternative procedures which provides an estimation of the dynamic behaviour. Among these, NORSOK N-003, ref /3/, states the "EDAF procedure", ref [4], as the required method to calculate the dynamic contribution of dynamic exposed structures such as Jackets (normally understood to have natural periods above 2-3 seconds). In short, the EDAF method is extracting the dynamics from a number (typically 30) of 3-hours second order time domain irregular sea states, solved by a non-linear structural solver. However, standard software packages typically do not have capability to generate the second order waves and solve the structure non-linearly. This work presents a procedure to perform investigations of the dynamic behaviour of jackups with software capable to generate first order random wave series and solve the structure as a dynamic analyse, with linear material and soil properties.
This procedure is intended to be used together with the global analysis procedures as stated in ref [2] to determine the dynamic effect (or inertia load) only. The other relevant loads such as global wave, current and wind loads should be additionally considered.

As time-domain analyses better capture the nature of the physical behaviour compared to SDOF estimations, the results reveal dependencies that will not be captured by traditional SDOF estimations. This is presented in the chapter 5.

2. SYMBOLS AND ABBREVIATIONS

An overview and explanation of the most used symbols and abbreviations is given below.

BS	Base Shear, the total shear force at the spudcan-soil interface
$BS_{dynamic}$	The dynamic notation is used both on the BS and OTM, meaning the dynamic response of the BS or OTM.

BS_{static}	The static notation is used both on the BS and OTM, meaning the static response of the BS or OTM. In this work these values are not taken from a static structural analysis, but the waveloads are used directly.
$EL_{inertia\ load}$	Elevation of where the inertia load is applied in the global analysis
OTM	Overturning Moment, the global moment around an axis at the centre of the three legs rotation point (typically mid of spudcan penetration)
p-delta	Additional global moment due to the horizontal deflection of the jackup hull. For details; ref [2], ch 4.4.7
SDOF	Single Degree Of Freedom: In this text used as DAF evaluated by considering the jackup as a SDOF system
TDAF	Time-domain Dynamic Amplification Factor
Tp	The period of the energy peak of the applied wave spectrum
T_0	Natural period of the jackup
VCG	Vertical Centre of Gravity
σ	Standard deviation

3. MODEL FUNDAMENTALS

The analyses are performed on several different drilling jackups and/or locations, which could be summarised with the properties in Table 1.

Table 1 jackup main properties

Property:	Value:
Elevated weight	25 000 – 32 000 tonnes
Leg design	x-braced, modelled by an equivalent leg
Water depth	110 – 140 meters (main cases, shallower in the sensitivity checks)
Natural period	6 – 11 seconds

3.1. HYDRODYNAMIC ENVIROMENT

The waves are generated by a number of Airy-waves added on top of each other, in a random process. The wave loads are calculated by Morison equation, taking into account the instantaneous wave crest elevation. Detailed description of how software Sesam Wajac generate random waves is given in ref [5].

The wave generation is initiated by a chosen number, which determines the exact composition of the combined waves, e.g. the seed number 1 will result in equal 3-hour seastate each time it is generated. Selection of another number will cause a completely different realisation. When a number of realisations/seeds are to be analysed, it is possible to actively chose the initiation number (e.g. in a row from 1 to 10) or draw a random value. During the screening phase, it is essential to capture the relative differences rather than the absolute values. Hence, the seed selection should be controlled and be equal in the various screening seastates. For the final analysis, the absolute value is of a higher importance and a rapid result convergence is favourable, hence the seeds should be initiated by random numbers. This is further showed in Ch 5.2.

Wajac have the possibility to apply Wheeler stretching of the wave particle velocities towards the wave crest, ref [5]. It is assumed this will give a more realistic representation of the wave loads. However, as base case in this procedure, it is chosen not to utilise the stretching as it is significantly increasing the CPU solving time and no consistent major impact is found on the results.

Table 2 Summary of applied seastates

Main Properties	Applied parameters
No of seeds	10 seeds in screening, 30 seeds in main analysis
Seastate duration	3 hours
Wave order	First order waves

Wave spectrum	JONSWAP (with $\sigma_A = 0.07$, $\sigma_B = 0.09$, γ=3.3) and long-crested waves
Current	Current profile applied
Analyse time step	~1/25 T_p and <1/10 T_0
Drag Coefficients	0.65 above MSL+2m and 1.05 below (as basis for the equivalent leg calculation)

3.2. FE MODEL

Figure 1 Equivalent leg model and hull

In the model intended for the time-domain analysis, the whole leg below bottom of hull level is replaced with an equivalent leg model, in order to reduce the members in the time-domain analysis and by such significantly reduce the CPU solving time. Especially the wave load calculation is driven by the number of submerged FE elements. The calculation of the equivalent leg is carried out in accordance with the procedure given in ref /2/, Appendix A, which preserves both the structural and hydrodynamic properties.

The leg to hull connections and the hull itself, are modelled as follows:

- The spudcans are modelled by fictitious stiff beam members with the main purpose to get a representation of the spudcan-to-leg interaction.
- The hull structure is modelled by horizontal beams in a “grid” as illustrated in Figure 3 1. The purpose is that the hull beam model is calibrated in order to represent the hull global stiffness. The vertical location of this grid is in the hull centre line elevation.
- The jacking house are represented as vertical beams connected to the hull, with node in between representing the leg-to-hull connections, i.e. lower guides, fixation systems, pinions and upper guides.
- The leg-to-hull connections are modelled by hinges such that only selected force components are transferred between the leg chords and the hull structures. Moments are not transferred at the leg-to-hull connections. The forces transferred are:

  Lower and upper guides: Horizontal forces only in the rack plane for each chord
  Fixation systems: Both horizontal and vertical forces
  Pinions: No forces transferred

Further essential structural properties are stated in Table 3

Table 3

Property	Value/description
Young's Modulus	$E = 2.1E^{11}$ N/m^2 (Typical steel value) as a linear material, no yield or hardening
Structural damping	Rayleigh proportional damping is applied in the analysis, based on 4% of critical damping and the 1st and 4th natural period.
Soil constraint	Linear springs in all 6 DoFs. Dynamic rotational stiffness used to tune the natural period

4. TIME-DOMAIN DYNAMIC AMPLIFICATION (TDAF) ANALYSES

The FE model as summarised above is used in the wave load analysis, where a number of the irregular linear seastates are applied. The structure is thereafter solved for the determined wave loads in a dynamic linear structural analysis. The final goal is to determine the characteristic difference between the static and the dynamic response of the jackup. As we consider only global effects herein, the wave load calculation provides sufficient data (base shear and overturning moment) to identify the static response. These values could then be compared with the results from the dynamic response and the effect of the dynamic is captured. In order substantiate the magnitude of the difference, sufficient number of realisations are required.

The TDAF procedure can be summarised with the following steps:

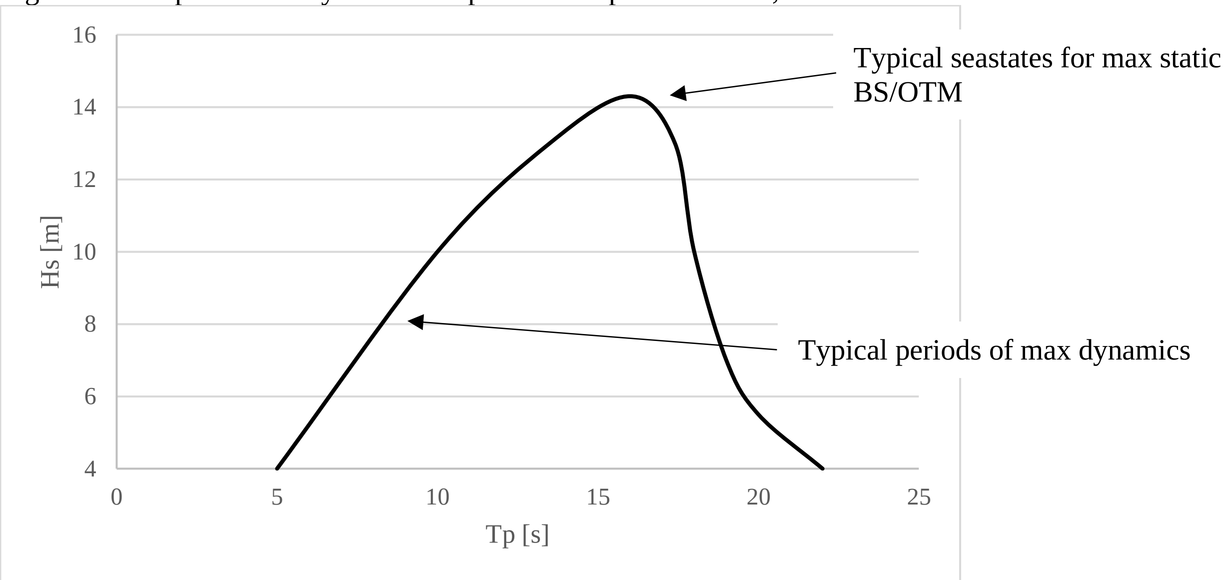
1. Screening
2. TDAF determination
3. Implementation in global analysis

4.1. SCREENING FOR GOVERNING SEASTATE

The wave loads on jackups are highly drag dominated, implying that the highest forces are given of the seastate with the highest wave particle velocities. For typical jackup sizes and in harsh environment (e.g. the North Sea) the seastates at the top and slightly to the right of the contour line normally give the highest wave loads. As mentioned above, the typical natural periods of these rigs are in the range of 6-12 seconds and the highest dynamic amplification is at the natural period, see illustration in Figure 2. How these two effects add to each other is subject to the particular case and it have not been possible to draw a clear guidance of which seastate is the governing. Hence, a screening along top of the contour is performed to identify the governing seastate with respect to global base shear and overturning moment.

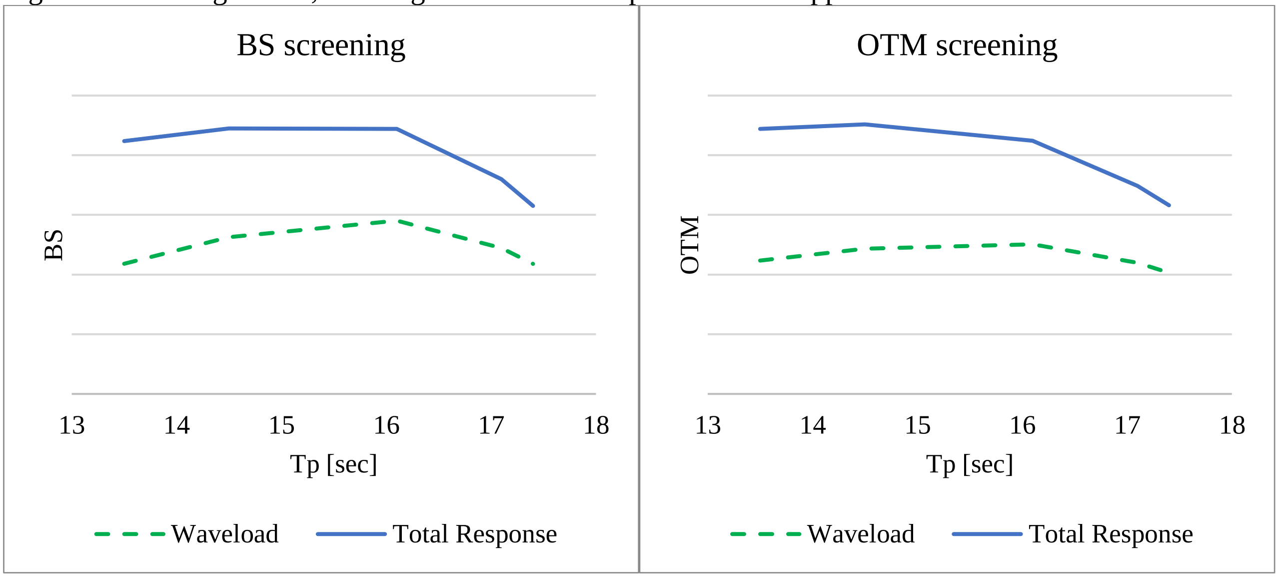
In order to have the screening seastates as comparable as possible, the seed initiation number is selected manually and repeated for each of the seastates.

Figure 2 Example of a 100 years return period Hs/Tp contour line, with additional considerations



The plots below show the results from such a screening process, using 5 seastates around the top as presented in Figure 2. The results show that the maximum base shear and overturning moment from the wave loads alone occurs at the contour top. However, including the dynamic response (labelled “Total response”) the governing seastate have shifted. In this case it is a lower Tp which is found governing, but also increased Tp is seen governing is other cases.

Figure 3 Screening results, showing both the total response and the applied waveload



4.2. TDAF DETERMINATION

4.3. POSTPROCESSING OF THE RESULTS

For each seed the base shear (BS) and overturning moment (OTM) are saved for every time step, both for the wave load and for the structural response. These time series are saved and postprocessed in order to calculate the TDAF, with the following steps:

1. The raw values are given in X and Y direction and are then calculated to a resultant BS and OTM.
2. The max BS and OTM for both the wave load and structural response are extracted for each seed.
3. To get the Gumbel distribution and linear fitting, the extracted maxima are sorted in an ascending order and the double negative logarithmic to the order number is calculated (Least square fitting), by the following formula:

   $$Y_{axis} = -\ln\left(-\ln\left({}^{no\,-\,0.44}\big/_{seeds\,+\,0.12}\right)\right) \qquad \text{(Eq. 1)}$$

   As an alternative, the Gumbel distribution can be also set based on the maxima standard deviation and average value:

   $$(\ \alpha = 0.7797\,\sigma \qquad \beta = avg - 0.57722 * \alpha\ ) \qquad \text{(Eq. 2)}$$

   The X-axis in the plot is then the magnitude of the response in question, e.g. the global base shear.
4. The maxima are linearized, and the ratio between structural response and the static wave load is calculated, on a selected percentile load level, both for BS and OTM. The percentile is selected so that the static BS is matching the deterministic BS obtained from the design wave.
5. At the given percentile, these values are extracted; $BS_{dynamic}$, BS_{static}, $OTM_{dynamic}$, OTM_{static}. With these the dynamic amplification factors are calculated:

   $$\mathrm{DAF}_{BS} = \frac{BS_{dynamic}}{BS_{static}} \qquad \text{(Eq. 3)}$$

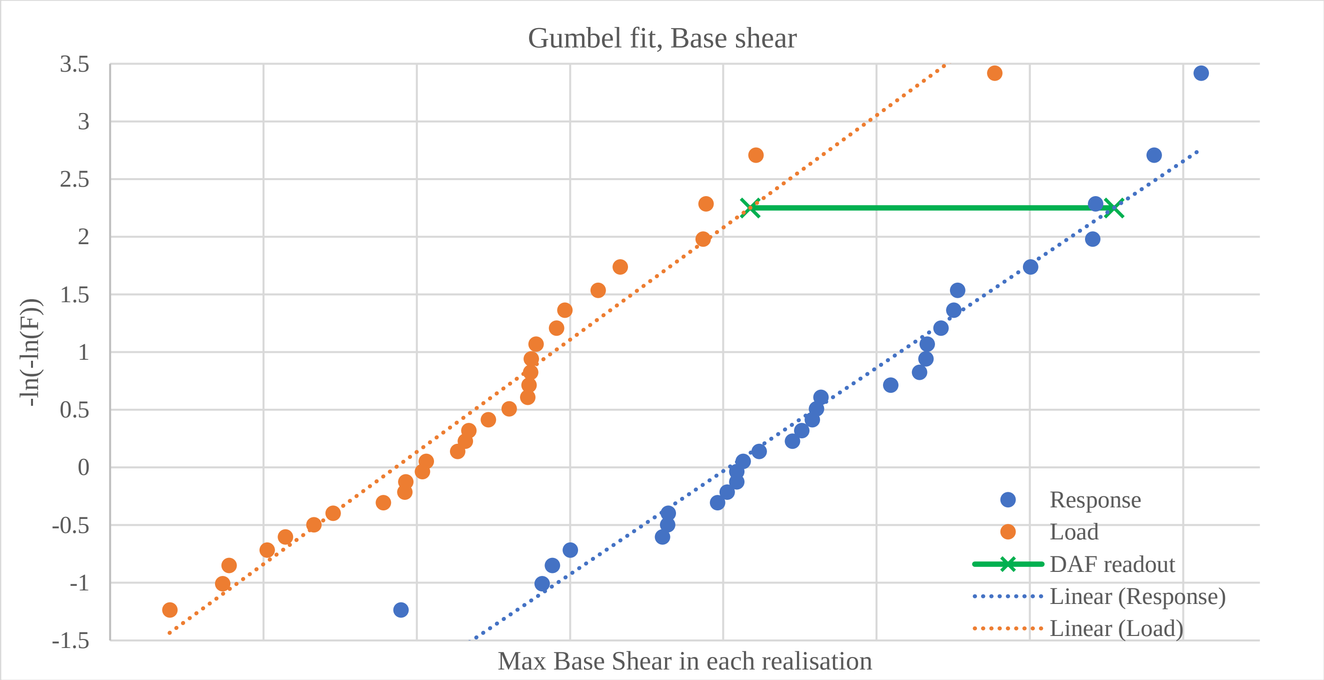
   $$\mathrm{DAF}_{OTM} = \frac{OTM_{dynamic}}{OTM_{static}} \qquad \text{(Eq. 4)}$$

6. As normally only one DAF is to be applied, as an added “inertia load” at the hull level, this should cover both the BS DAF and the OTM DAF. Then the representative inertia load is tuned in order to match them both, by this equation:

   $$DAF_{tuned} = 1 + \frac{OTM_{Dyn} - OTM_{static}}{EL_{inertia\ load} * BS_{static}} \qquad \text{(Eq. 5)}$$

   As a result, the tuned DAF will be typically higher than the BS DAF and lower than OTM DAF, i.e., the applied global shear will be increased on the conservative side. Alternatively, it may be possible to split the BS and OTM in two, as separate additional inertia load and a moment, to get more accurate representation of the dynamic effect.

Figure 4 Example of the DAF readout, based on Gumbel fitted maxima



4.4. IMPLEMENTATION IN GLOBAL ANALYSIS

The calculated inertia effects in sec 4.3 is to be conserved in the strength analyses. Normally this is represented as an inertia load, based on one amplification factor to the static BS. For well calibrated time-domain analyses and where the readout percentile is adjusted to the design wave, also the absolute inertia values may be suitable to use. If required to optimise the utilisation checks, it is also possible to refine the inertia load by applying the OTM dynamics as a separate inertia moment in addition to BS DAF. The implementation in global analysis is not presented herein.

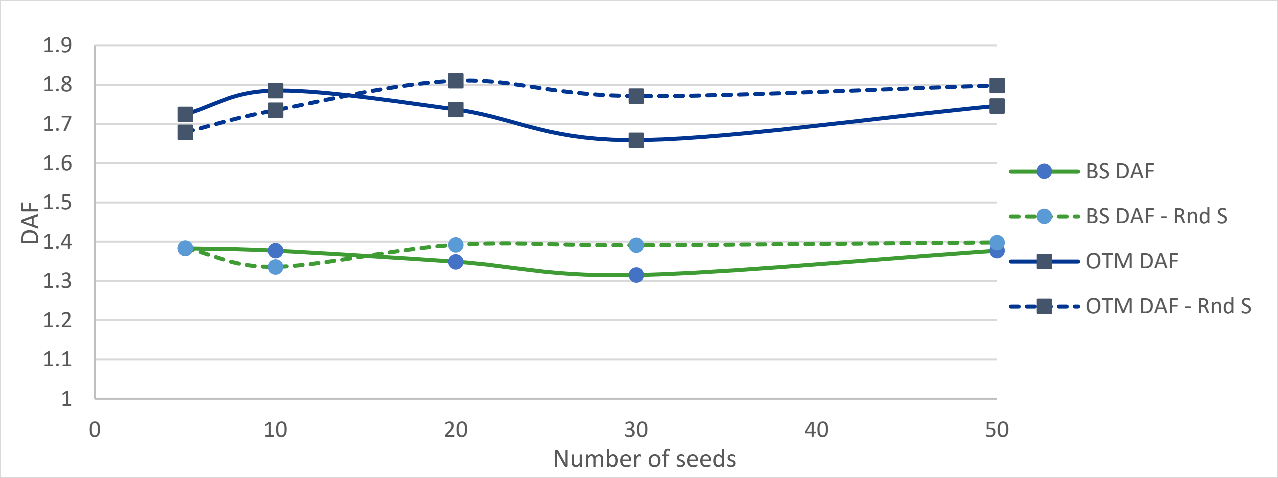
The dynamics calculated from time-domain analysis is normally based on the total response and the maximum wave load, and results in an amplification factor between these two parameters. However, the DAF calculated from SDOF method is an amplification of the load *amplitude*, not the absolute maximum. Hence, the magnitudes of TDAF and SDOF DAF cannot be directly compared with each other as they are intended to scale different basis. This becomes especially important if current is included in the analysis, as this will further shift the oscillation centre.

5. INPUT VARIATIONS

5.1. SEED CONVERGENCE

In order to check the result convergence as function of the number of realisations, two different analyses setups where checked, one with determined seed initiation number and one with random numbers. Both are then solved with 5, 10, 20, 30 and 50 seeds. For the ones with determined numbers, the same realisations are reused as the 5 seed check have the seed number 1,2,3,4, 5 and the 50 seed check have the numbers from 1 to 50. For the random selection it is likely that very few of the same realisations are drawn in the various checks. The results for base shear and overturning DAF are presented in Figure 5.

Figure 5 Seed convergence plot

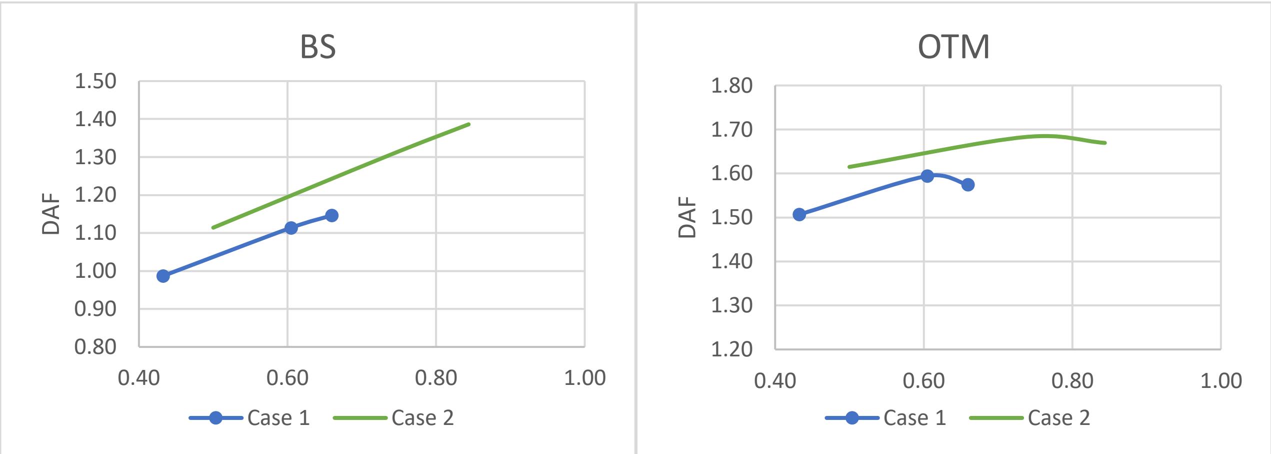


The plot above show that the random seed selection (denoted "Rnd S" in the plot) does converge faster than the determined seeds. 20 realisations seem to give sufficient accuracy, but if the seed number is determined, 50 seeds or more is required to identify the converged solution.

5.2. IMPACT OF WATER DEPTH

The dynamics of a jackup may be compared with free vibration of a cantilever beam with a lumped mass at free end. The waveloads acts significantly below this lumped mass. In order to study the effect of the water depth, i.e. the load centre caused by waves, two different cases were analysed by the TDAF procedure. In this assessment various water depths are applied but all other parameters remain unchanged. The current is not included. The following two plots show the DAF versus ratio of water depth/ hull VCG.

Figure 6 DAF in different water depth/hull VCG ratios



The plots give a clear indication that the ratio does impact the dynamic behaviour, especially for the base shear DAF. This effect might be understood by the lumped mass and beam analogy. If the centre of applied forces (waveload) is close to the oscillating mass, the amplification increases. Opposite, if the load is closer to the fixed support, the dynamic effect decreases.

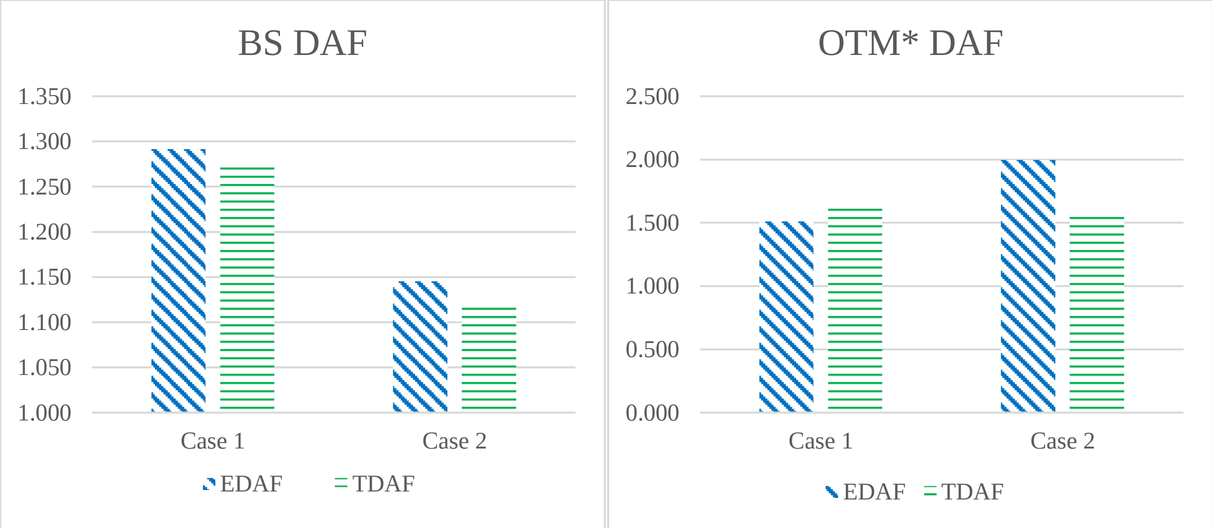
This effect indicates that increasing the airgap at a certain site might be a beneficial, as the dynamic behaviour will be reduced. However, there are other effects, as wind and p-delta, which will counteract the reduced dynamic and it is not obvious which ones are prevailing.

6. COMPARISON TOWARDS OTHER METHODS

6.1. EDAF COMPARISON

The TDAF procedure where run for two units/sites where comparable EDAF results where available to compare the result. It is to be noted that the EDAF analyses is performed on other software and there might be properties which are not fully aligned These EDAF analysis do utilise a second order wave model, which will give more realistic and steeper wave crests than the linear waves in the TDAF analysis. The BS results in Figure 7 might indicate that such waves give a slightly higher amplification, as the EDAF are larger than the TDAF results. However, the differences are considered minor.

Figure 7 Dynamic amplification factor comparison between EDAF and TDAF results

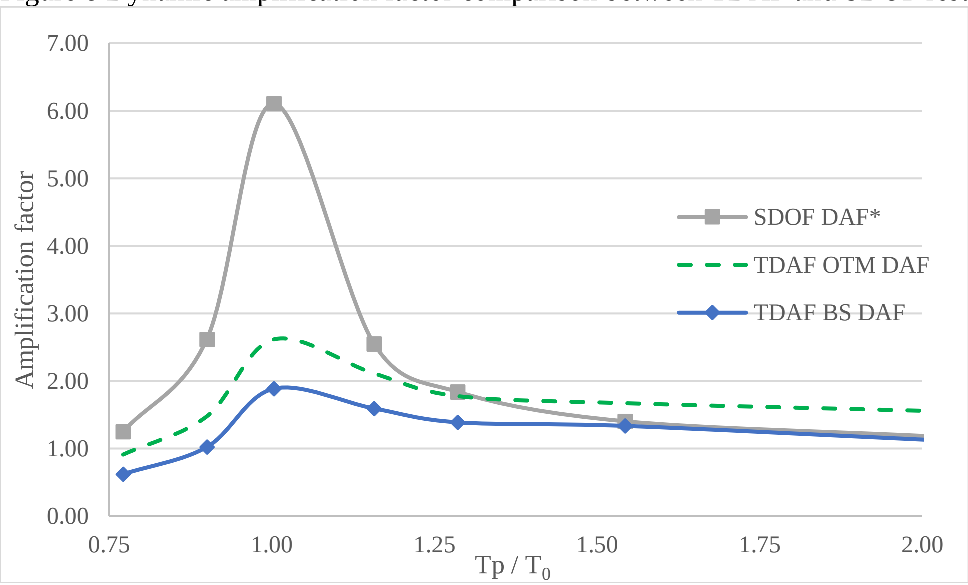


* OTM DAF results are not fully comparable. As the structural non-linear behaviour is taken into account in the EDAF analysis, the OTM result do also include part of the p-delta effect in its nature. This is not included in the TDAF analysis and the p-delta will be added afterwards in the global analysis.

6.2. SDOF COMPARISON

Often the dynamic effect is estimated by considering the jackup as a SDOF and calculate the amplification based on the ratio between the seastate peak period vs jackup natural period and include the effect of damping. This method gives high amplifications where the mentioned ratio is close to 1 (infinite at 1 with no damping) and it is only the damping which give a reduction. In Figure 8 SDOF DAF* values are compared to amplification factors from TDAF.

Figure 8 Dynamic amplification factor comparison between TDAF and SDOF results



* SDOF DAF results are modified to improve the comparability towards the TDAF magnitudes, by these means:

- According to ref [2] the T should be taken as 0.9 x Tp. This is not done and would move the SDOF peak slightly to the right in the plot (at ~1.1).
- Since TDAF and SDOF should be applied to different basis (amplitude vs total, see also ch 4.4) these values have been reduced by this formula (DAF-1) * 0.57 + 1 which is valid for this particular case, but not as a generic formulation.

The plot show that the high SDOF peak is reduced with the TDAF method and the base shear TDAF results aligns with higher ratios. Also considering the overturning moment amplification, this one has higher magnitudes in parts of the plot. It is to be noted, that in some cases we have seen a new small peak for the TDAF curves around $Tp/T_0 = 2$, meaning that the waves hit every second natural oscillation.

7. SUMMARY

The TDAF procedure is found efficient to run in the DNV Sesam package and the irregular random seastates gives a more realistic understanding of the dynamic behaviour. The improved representation of the global response in an irregular seastate, show a more "even" amplification factor than compared by the DAF given by SDOF considerations. For natural period close to the wave peak period, the TDAF evaluated amplifications are decreased, but might be larger than the SDOF ones when the wave periods become much larger than the natural periods of the rig.

The presented method is tuned and found acceptable in the aim of providing added knowledge about the ratio between the static loads and dynamic responses of typical jackups, where this ratio is used in a traditional deterministic global analysis in accordance with [2]. In such global analysis, the static wave load is calculated by a deterministic wave load analysis, e.g. by a 5^{th} order design Stoke wave. This work is not intended to challenge the design wave loads from such wave. That is also the reason for adjusting the percentile for the DAF readout in the result distribution towards this mentioned design wave. In the implementation of the found DAF, it is of outmost importance to conserve the absolute value of the inertia load from the TDAF analysis. As the SDOF evaluations and time-domain based calculations are based on the *amplitude* and *total* wave load, respectively, the amplification magnitude cannot be compared to each other. In this work, the philosophy of the seed initiation number selection is shown to be of importance. A random selection seems to get converged results with fewer seeds or realisations than if the initiation number is selected. On the other side, selected numbers improve the comparability between different seastates and by such superior in screening purposes.

In general, the presented method is considered suitable and gives valuable insight of the dynamic behaviour which could be included in the typical global analysis with small efforts.

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